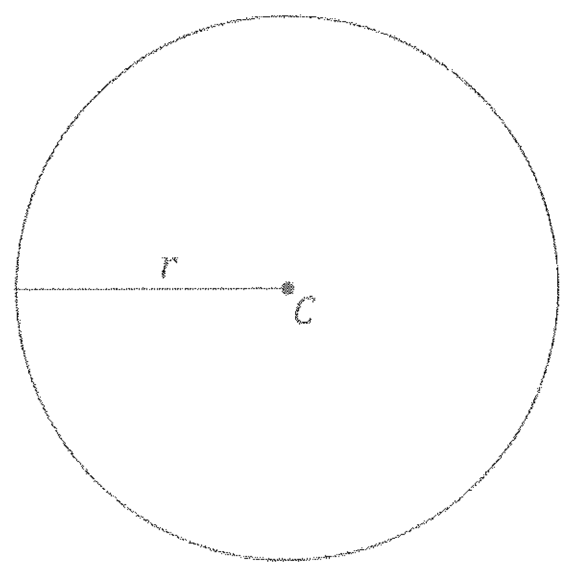
Homework Helpers

Geometry
Module 1

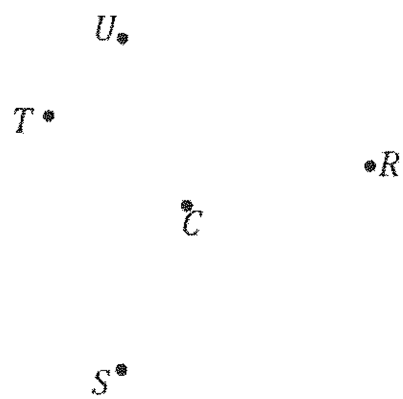
Lesson 1: Construct an Equilateral Triangle

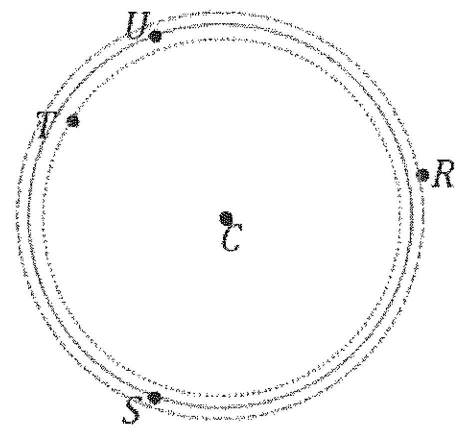
1. The segment below has a length of r. Use a compass to mark all the points that are at a distance r from point C.

> I remember that the figure formed by the set of all the points that are a fixed distance from a given point is a circle.



2. Use a compass to determine which of the following points, R, S, T, and U, lie on the same circle about center C. Explain how you know.



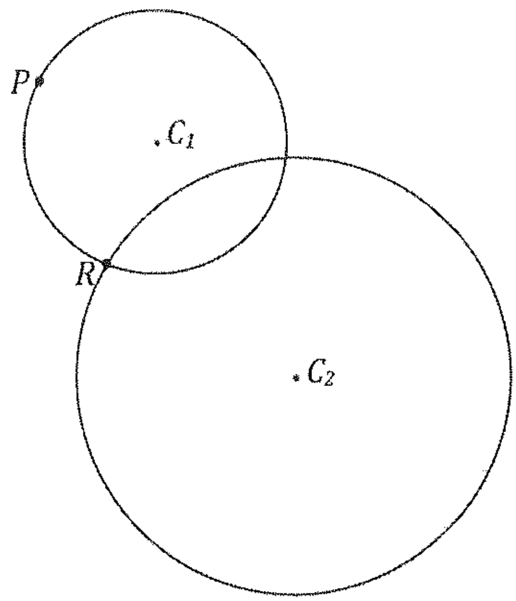


If I set the compass point at C and adjust the compass so that the pencil passes through each of the points R, S, T, and U, I see that points S and U both lie on the same circle. Points R and T each lie on a different circle that does not pass through any of the other points.

Another way I solve this problem is by thinking about the lengths CR, CS, CU, and CT as radii. If I adjust my compass to any one of the radii, I can use that compass adjustment to compare the other radii. If the distance between any pair of points was greater or less than the compass adjustment, I would know that the point belonged to a different circle.

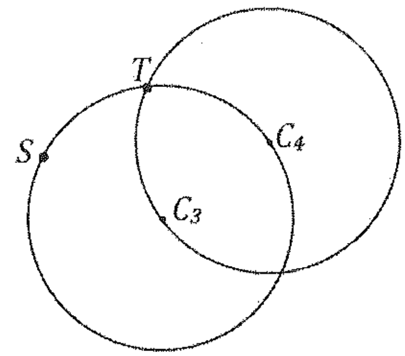
3. Two points have been labeled in each of the following diagrams. Write a sentence for each point that describes what is known about the distance between the given point and each of the centers of the circles.

a. Circle C_1 has a radius of 4; Circle C_2 has a radius of 6.



Point P is a distance of 4 from C_1 and a distance greater than 6 from C_2. Point R is a distance of 4 from C_1 and a distance 6 from C_2.

b. Circle C_3 has a radius of 4; Circle C_4 has a radius of 4.



Since the labeled points are each on a circle, I can describe the distance from each point to the center relative to the radius of the respective circle.

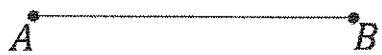
Point S is a distance of 4 from C_3 and a distance greater than 4 from C_4. Point T is a distance of 4 from C_3 and a distance of 4 from C_4.

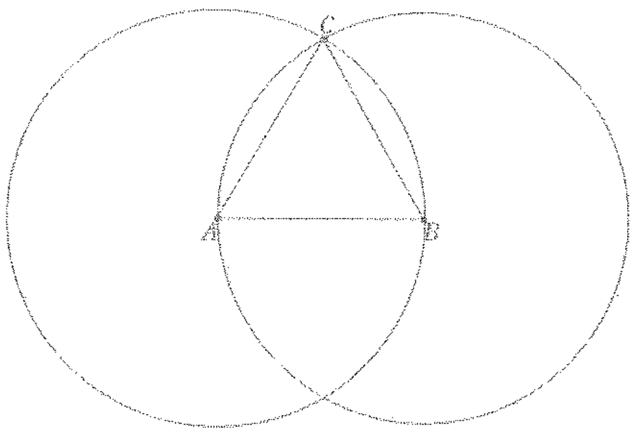
c. Asha claims that the points C_1, C_2, and R are the vertices of an equilateral triangle since R is the intersection of the two circles. Nadege says this is incorrect but that C_3, C_4, and T are the vertices of an equilateral triangle. Who is correct? Explain.

Nadege is correct. Points C_1, C_2, and R are not the vertices of an equilateral triangle because the distance between each pair of vertices is not the same. The points C_3, C_4, and T are the vertices of an equilateral triangle because the distance between each pair of vertices is the same.

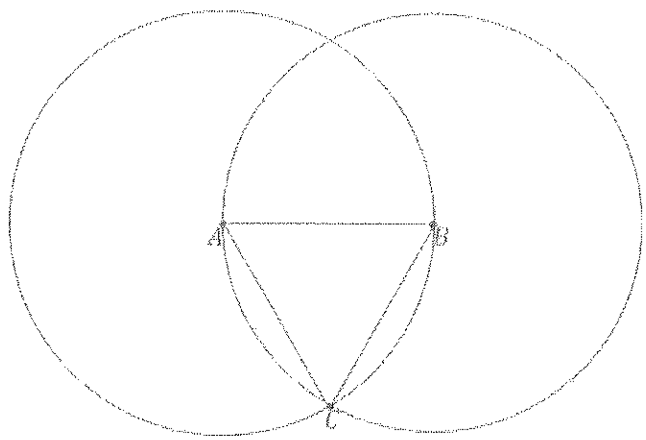
4. Construct an equilateral triangle ABC that has a side length AB, below. Use precise language to list the steps to perform the construction.

I must use both endpoints of the segment as the centers of the two circles I must construct.





or



1. *Draw circle A: center A, radius AB.*
2. *Draw circle B: center B, radius BA.*
3. *Label one intersection as C.*
4. *Join A, B, C.*

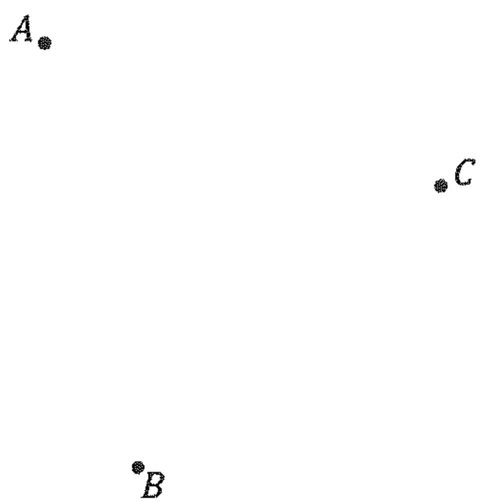
EUREKA MATH™

Lesson 2: Construct an Equilateral Triangle

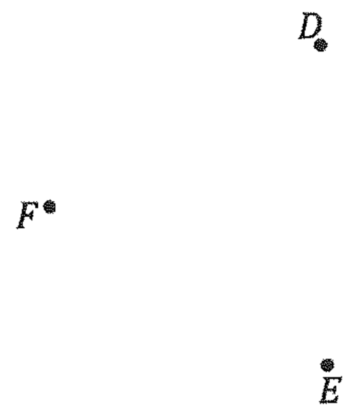
1. In parts (a) and (b), use a compass to determine whether the provided points determine the vertices of an equilateral triangle.

> The distance between each pair of vertices of an equilateral triangle is the same.

a.



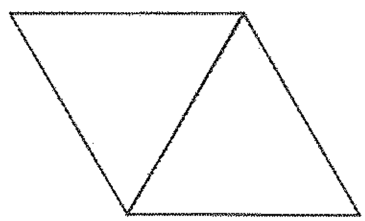
b.

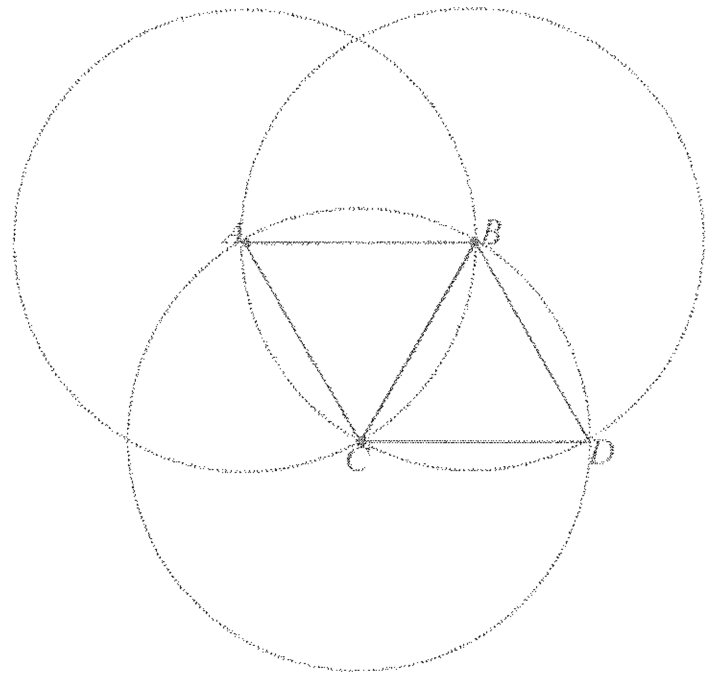


Points A, B, and C do not determine the vertices of an equilateral triangle because the distance between A and B, as measured by adjusting the compass, is not the same distance as between B and C and as between C and A.

Points D, E, and F do determine the vertices of an equilateral triangle because the distance between D and E is the same distance as between E and F and as between F and D.

2. Use what you know about the construction of an equilateral triangle to recreate parallelogram $ABCD$ below, and write a set of steps that yields this construction.

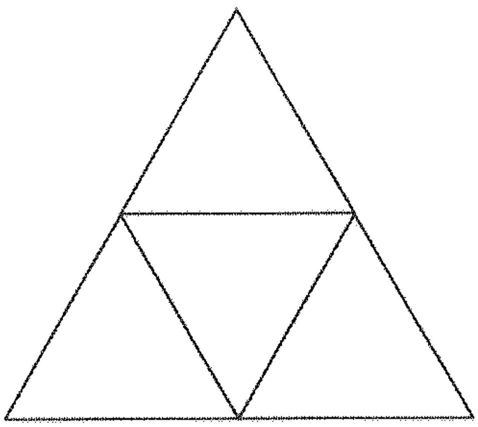


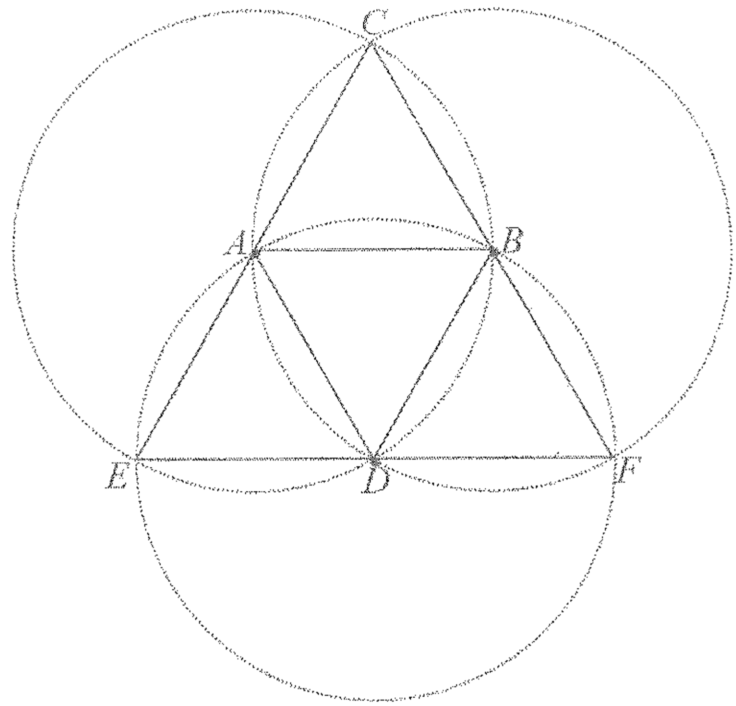


Possible steps:

1. *Draw* $\overline{AB}$.
2. *Draw circle* A: *center* A, *radius* AB.
3. *Draw circle* B: *center* B, *radius* BA.
4. *Label one intersection as* C.
5. *Join* C *to* A *and* B.
6. *Draw circle* C: *center* C, *radius* CA.
7. *Label the intersection of circle* C *with circle* B *as* D.
8. *Join* D *to* B *and* C.

3. Four identical equilateral triangles can be arranged so that each of three of the triangles shares a side with the remaining triangle, as in the diagram. Use a compass to recreate this figure, and write a set of steps that yields this construction.





Possible steps:

1. *Draw $\overline{AB}$.*
2. *Draw circle A: center A, radius AB.*
3. *Draw circle B: center B, radius BA.*
4. *Label one intersection as C; label the other intersection as D.*
5. *Join A and B with both C and D.*
6. *Draw circle D: center D, radius DA.*
7. *Label the intersection of circle D with circle A as E.*
8. *Join E to A and D.*
9. *Label the intersection of circle D with circle B as F.*
10. *Join F to B and D.*

Lesson 3: Copy and Bisect an Angle

1. Krysta is copying $\angle ABC$ to construct $\angle DEF$.

I must remember how the radius of each of the two circles used in this construction impacts the key points that determine the copied angle.

   a. Complete steps 5–9, and use a compass and straightedge to finish constructing $\angle DEF$.

   Steps to copy an angle are as follows:

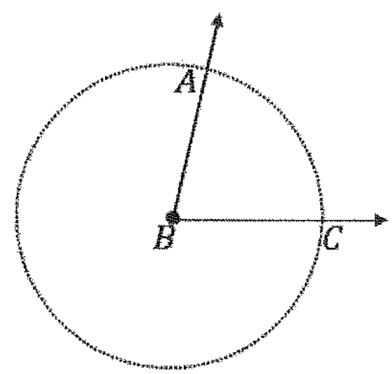
   1. Label the vertex of the original angle as B.
   2. Draw $\overrightarrow{EG}$ as one side of the angle to be drawn.
   3. Draw circle B: center B, any radius.
   4. Label the intersections of circle B with the sides of the angle as A and C.
   5. Draw circle E: center E, radius BA.

   6.6 Label intersection of circle with $\overrightarrow{EG}$ as F.

   7.7 Draw circle F: center F, radius CA.

   8.8 Label either intersection of circle E and circle F as D.

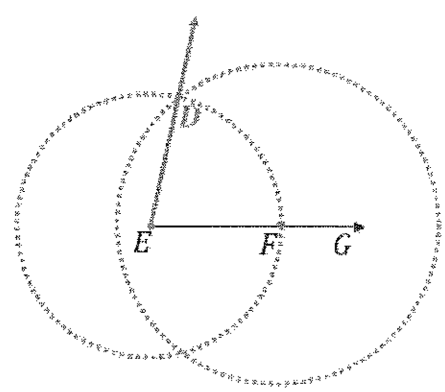
   9. Draw $\overrightarrow{ED}$.



   b. Underline the steps that describe the construction of circles used in the *copied* angle.

   c. Why must circle F have a radius of length CA?

   The intersection of circle B and circle C: center C, radius CA determines point A. To mirror this location for the copied angle, circle F must have a radius of length CA.



EUREKA MATH

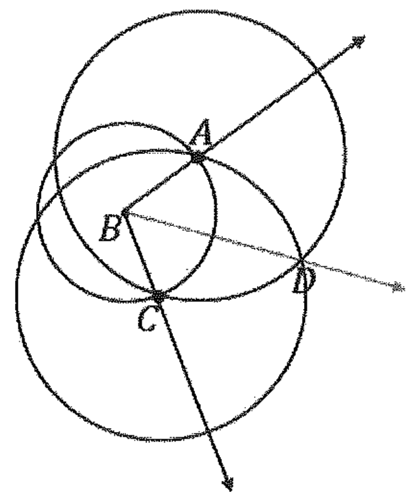
2. $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$.

   a. Write the steps that yield $\overrightarrow{BD}$ as the angle bisector of $\angle ABC$.

   I have to remember that the circle I construct with center at B can have a radius of any length, but the circles with centers A and C on each of the rays must have a radius AC.

   Steps to construct an angle bisector are as follows:

   1. ***Label the vertex of the angle as B.***
   2. ***Draw circle B: center B, any size radius.***
   3. ***Label intersections of circle B with the rays of the angle as A and C.***
   4. ***Draw circle A: center A, radius AC.***
   5. ***Draw circle C: center C, radius CA.***
   6. ***At least one of the two intersection points of circle A and circle C lies in the interior of the angle. Label that intersection point D.***
   7. ***Draw*** $\overrightarrow{BD}$.



   b. Why do circles A and C each have a radius equal to length AC?

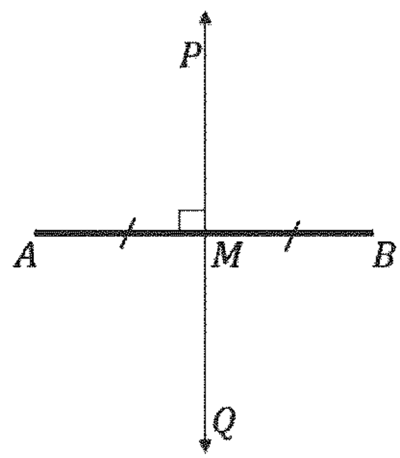
   Point D is as far from A as it is from C since $AD = CD = AC$. ***As long as A and C are equal distances from vertex B and each of the circles has a radius equal AC, D will be an equal distance from*** $\overrightarrow{AB}$ ***and*** $\overrightarrow{AC}$. ***All the points that are equidistant from the two rays lie on the angle bisector.***

Lesson 4: Construct a Perpendicular Bisector

1. Perpendicular bisector $\overleftrightarrow{PQ}$ is constructed to $\overline{AB}$; the intersection of $\overleftrightarrow{PQ}$ with the segment is labeled M. Use the idea of folding to explain why A and B are symmetric with respect to $\overleftrightarrow{PQ}$.

> To be symmetric with respect to $\overleftrightarrow{PQ}$, the portion of $\overline{AB}$ on one side of $\overleftrightarrow{PQ}$ must be mirrored on the opposite side of $\overleftrightarrow{PQ}$.

If the segment is folded along $\overleftrightarrow{PQ}$ so that A coincides with B, then $\overline{AM}$ coincides with $\overline{BM}$, or $AM = BM$; M is the midpoint of the segment. $\angle AMP$ and $\angle BMP$ also coincide, and since they are two identical angles on a straight line, the sum of their measures must be $180°$, or each has a measure of $90°$. Thus, A and B are symmetric with respect to $\overleftrightarrow{PQ}$.

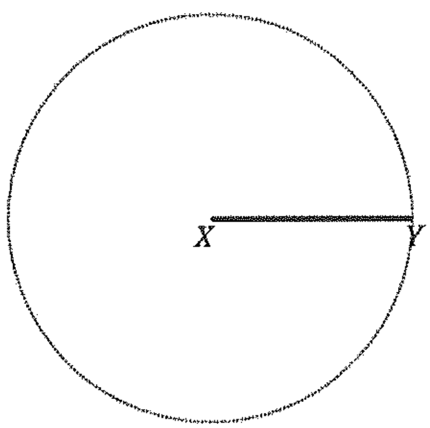


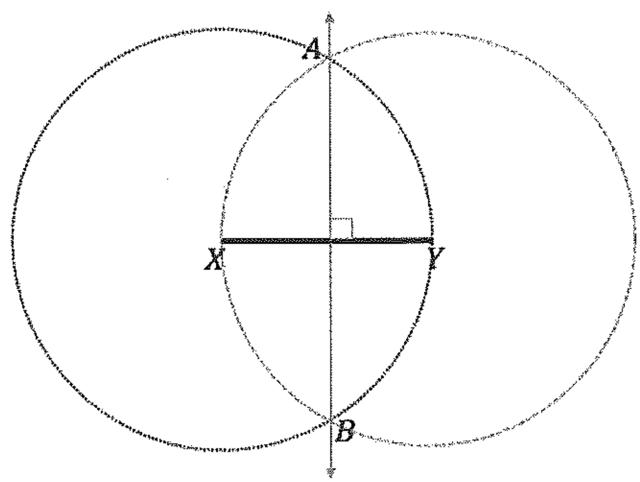
EUREKA MATH

2. The construction of the perpendicular bisector has been started below. Complete both the construction and the steps to the construction.

   1. Draw circle X: center X, radius XY.
   2. *Draw circle Y: center Y, radius YX.*
   3. *Label the points of intersections as A and B.*
   4. *Draw $\overleftrightarrow{AB}$.*

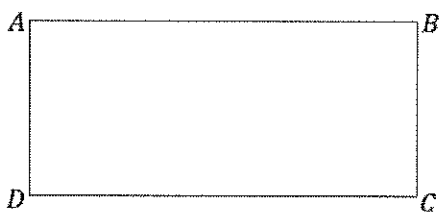
This construction is similar to the construction of an equilateral triangle. Instead of requiring one point that is an equal distance from both centers, this construction requires two points that are an equal distance from both centers.

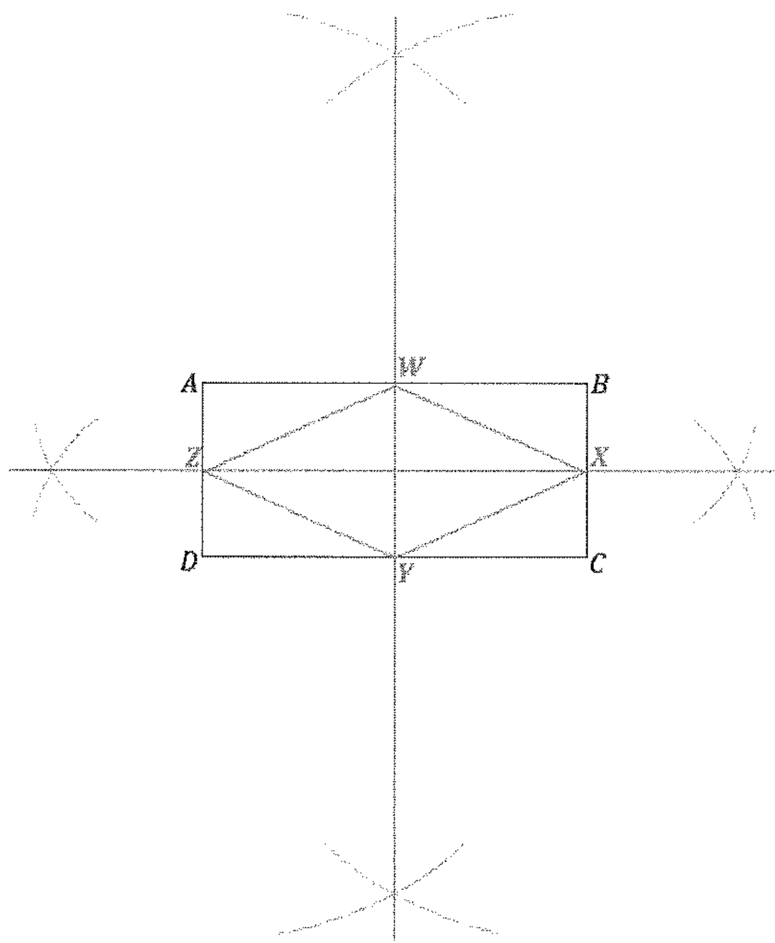




3. Rhombus $WXYZ$ can be constructed by joining the midpoints of rectangle $ABCD$. Use the perpendicular bisector construction to help construct rhombus $WXYZ$.

The midpoint of $\overline{AB}$ is vertically aligned to the midpoint of $\overline{DC}$. I can use the construction of the perpendicular bisector to determine the perpendicular bisector of $\overline{DC}$.



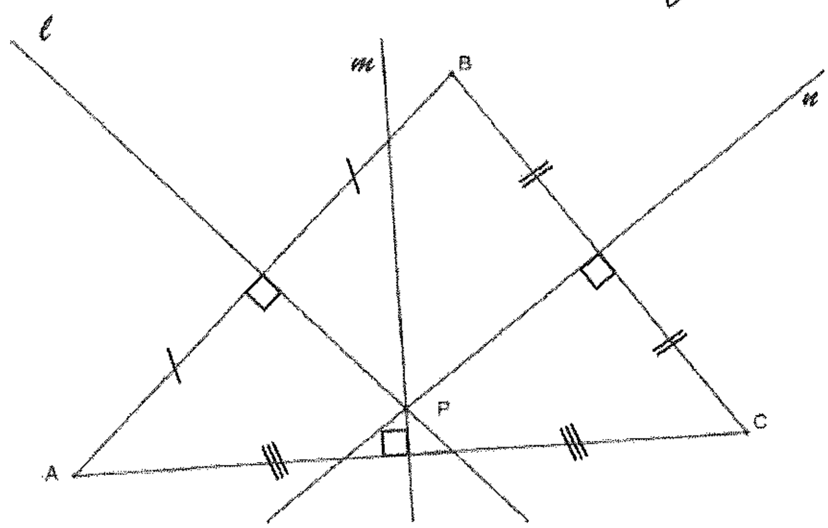


Lesson 5: Points of Concurrencies

1. Observe the construction below, and explain the significance of point P.

   Lines l, m, and n, which are each perpendicular bisectors of a side of the triangle, are concurrent at point P.

   The markings in the figure imply lines l, m, and n are perpendicular bisectors.



   a. Describe the distance between A and P and between B and P. Explain why this true.

   P is equidistant from A and B. Any point that lies on the perpendicular bisector of $\overline{AB}$ is equidistant from either endpoint A or B.

   b. Describe the distance between C and P and between B and P. Explain why this true.

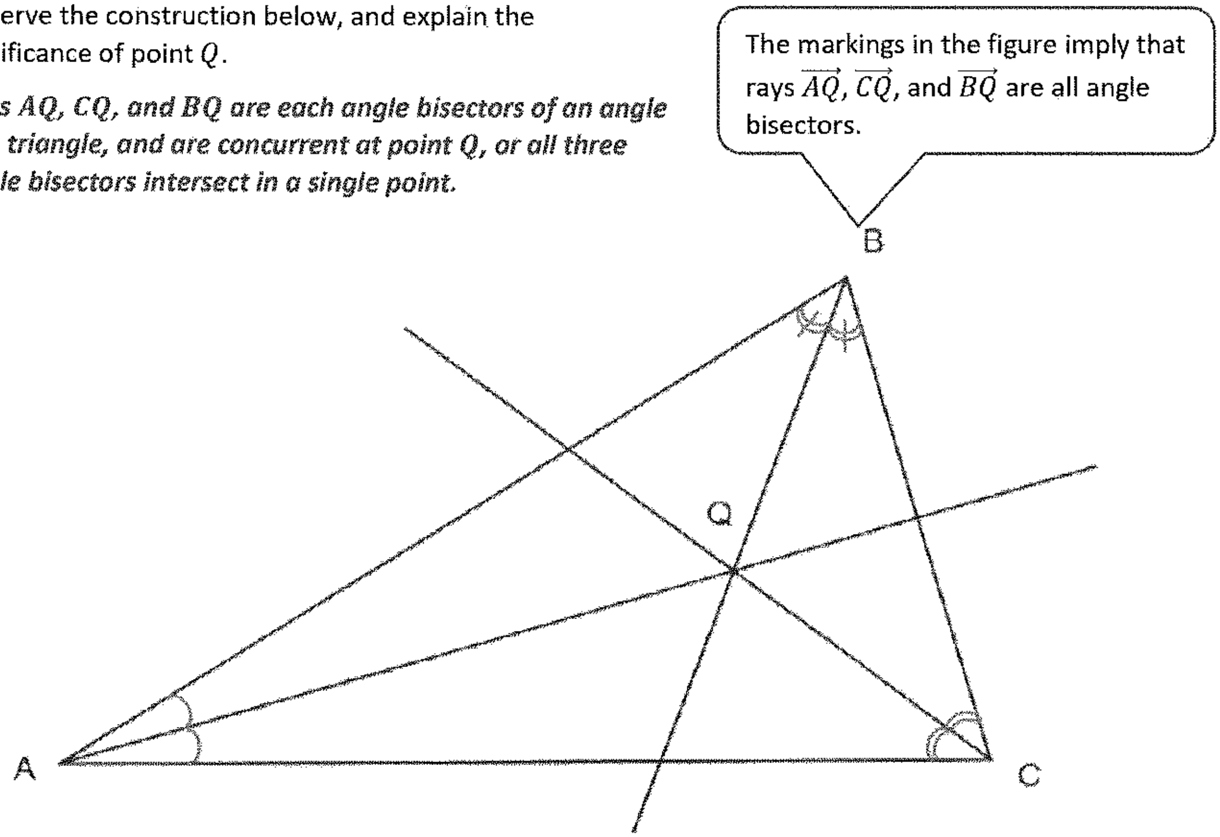
   P is equidistant from B and C. Any point that lies on the perpendicular bisector of $\overline{BC}$ is equidistant from either endpoint B or C.

   c. What do the results of Problem 1 parts (a) and (b) imply about P?

   Since P is equidistant from A and B and from B and C, then it is also equidistant from A and C. This is why P is the point of concurrency of the three perpendicular bisectors.

2. Observe the construction below, and explain the significance of point Q.

   Rays AQ, CQ, and BQ are each angle bisectors of an angle of a triangle, and are concurrent at point Q, or all three angle bisectors intersect in a single point.



   a. Describe the distance between Q and rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Explain why this true.

      Q is equidistant from $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Any point that lies on the angle bisector of $\angle A$ is equidistant from rays $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

   b. Describe the distance between Q and rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Explain why this true.

      Q is equidistant from $\overrightarrow{BA}$ and $\overrightarrow{BC}$. Any point that lies on the angle bisector of $\angle B$ is equidistant from rays $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

   c. What do the results of Problem 2 parts (a) and (b) imply about Q?

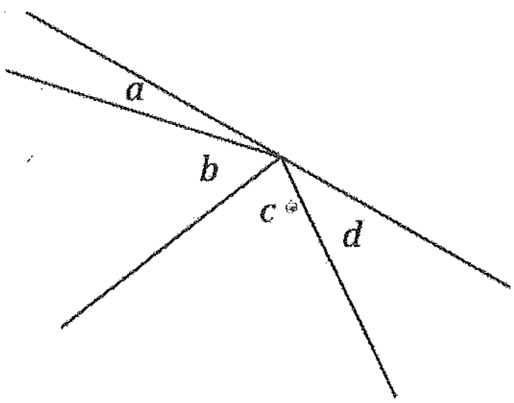
      Since Q is equidistant from $\overrightarrow{AB}$ and $\overrightarrow{AC}$ and from $\overrightarrow{BA}$ and $\overrightarrow{BC}$, then it is also equidistant from $\overrightarrow{CB}$ and $\overrightarrow{CA}$. This is why Q is the point of concurrency of the three angle bisectors.

Lesson 6: Solve for Unknown Angles—Angles and Lines at a Point

1. Write an equation that appropriately describes each of the diagrams below.

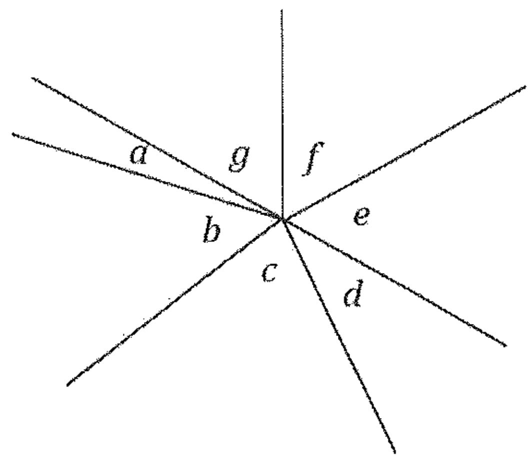
Adjacent angles on a line sum to 180°. Adjacent angles around a point sum to 360°.

a.



$a + b + c + d = 180°$

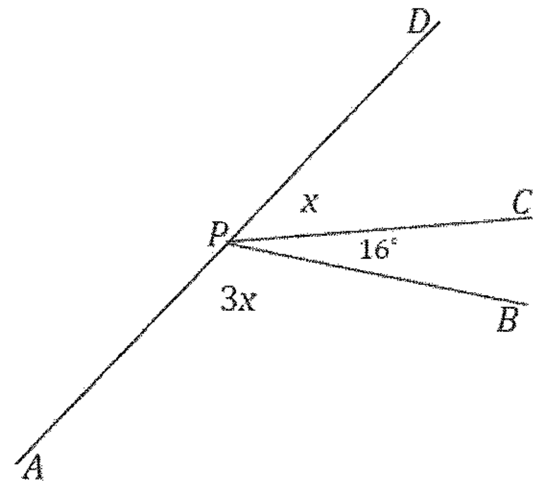
b.



$a + b + c + d + e + f + g = 360°$

2. Find the measure of $\angle APB$.

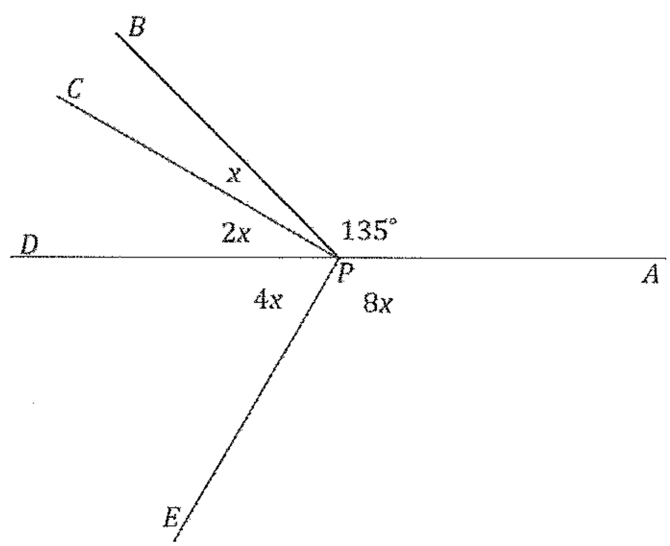
I must solve for x before I can find the measure of $\angle APB$.



$$3x + 16° + x = 180°$$
$$4x + 16° = 180°$$
$$4x = 164°$$
$$x = 41°$$

The measure of $\angle APB$ is $3(41°)$, or $123°$.

3. Find the measure of $\angle DPE$.



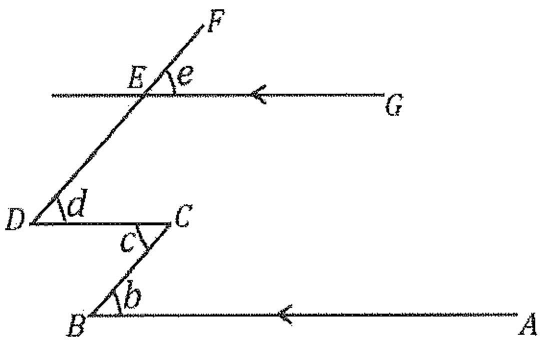
$$x + 2x + 4x + 8x + 135° = 360°$$
$$15x + 135° = 360°$$
$$15x = 225°$$
$$x = 15°$$

The measure of $\angle DPE$ is $4(15°)$, or $60°$.

Lesson 7: Solve for Unknown Angles—Transversals

1. In the following figure, angle measures b, c, d, and e are equal. List four pairs of parallel lines.

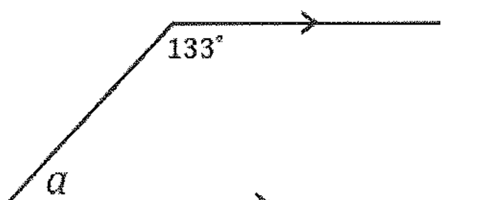
I can look for pairs of alternate interior angles and corresponding angles to help identify which lines are parallel.

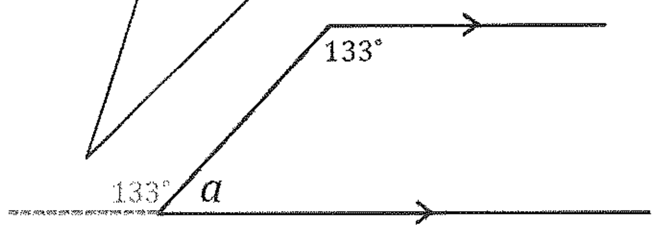


Four pairs of parallel lines: $\overleftrightarrow{AB} \parallel \overleftrightarrow{EG}$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{BC} \parallel \overleftrightarrow{DF}$, $\overleftrightarrow{CD} \parallel \overleftrightarrow{EG}$

2. Find the measure of ∠a.

I can extend lines to make the angle relationships more clear. I then need to apply what I know about alternate interior and supplementary angles to solve for a.

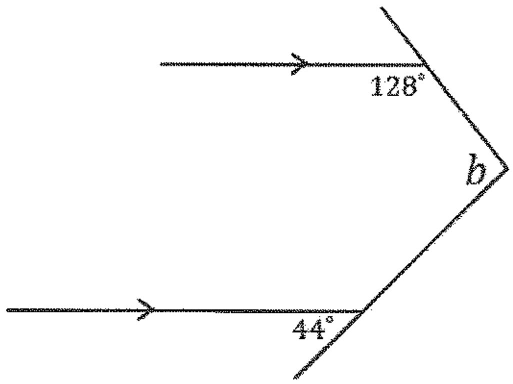


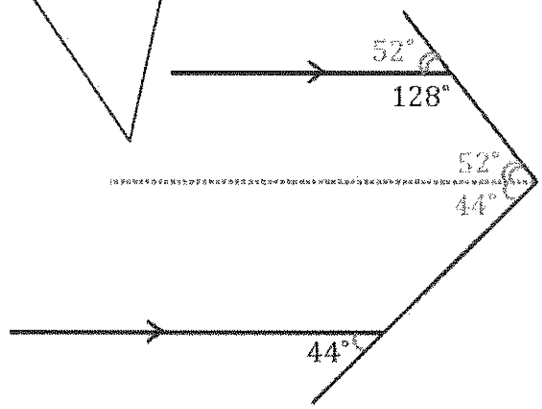


The measure of a *is* $180° - 133°$, *or* $47°$.

3. Find the measure of b.

I can draw a horizontal auxiliary line, parallel to the other horizontal lines in order to make the necessary corresponding angle pairs more apparent.

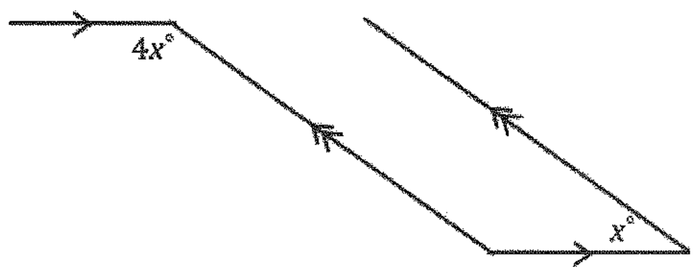


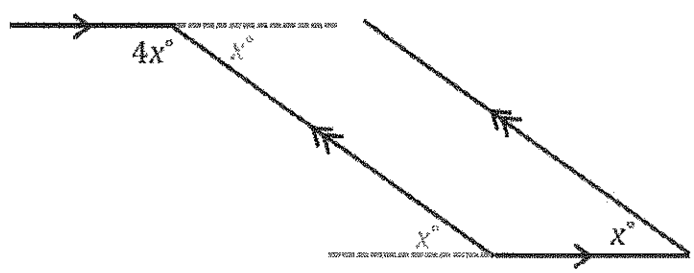


The measure of b is $52° + 44°$*, or* $96°$*.*

4. Find the value of x.

$$
\begin{aligned}
4x + x &= 180 \\
5x &= 180 \\
x &= 36
\end{aligned}
$$

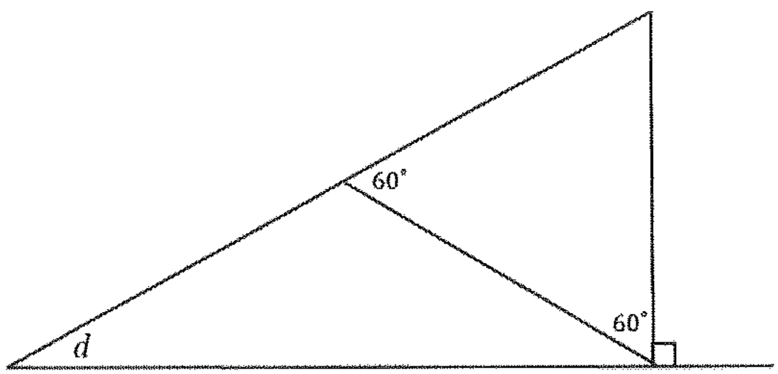


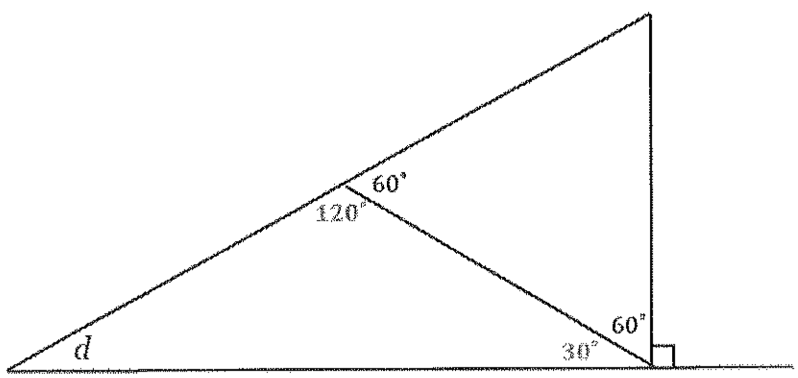


Lesson 8: Solve for Unknown Angles—Angles in a Triangle

1. Find the measure of d.

I need to apply what I know about complementary and supplementary angles to begin to solve for d.





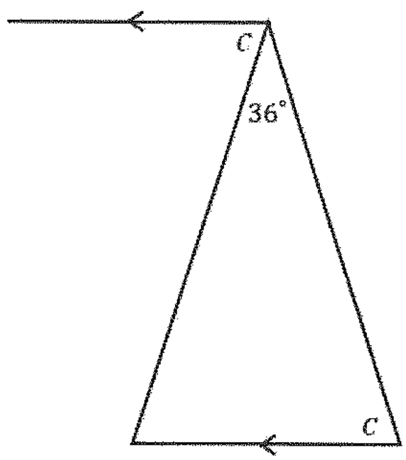
$$d + 120° + 30° = 180°$$

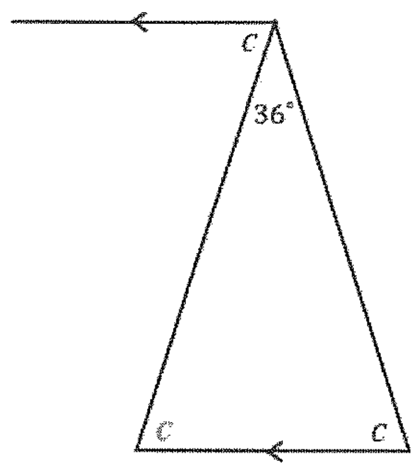
$$d + 150° = 180°$$

$$d = 30°$$

2. Find the measure of c.

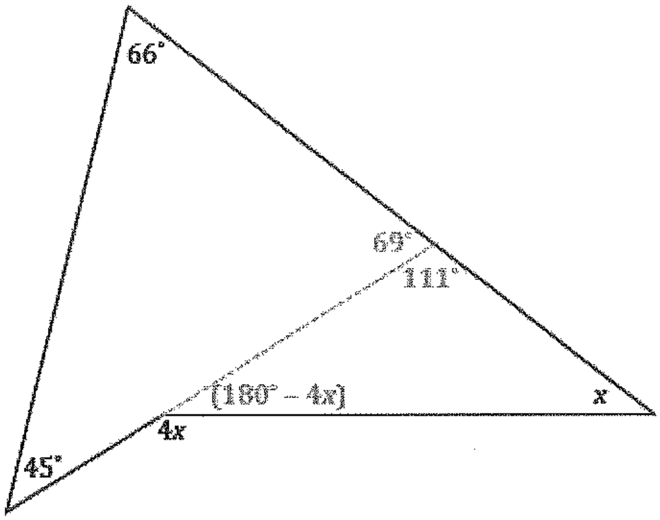
I need to apply what I know about parallel lines cut by a transversal and alternate interior angles in order to solve for c.





$$2c + 36^\circ = 180^\circ$$
$$2c = 144^\circ$$
$$c = 72^\circ$$

3. Find the measure of x.



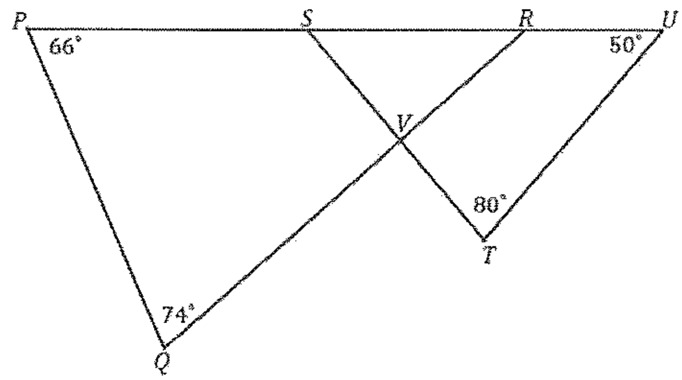
I need to add an auxiliary line to modify the diagram; the modified diagram has enough information to write an equation that I can use to solve for x.

$$\begin{aligned}(180^\circ - 4x) + 111^\circ + x &= 180^\circ\\ 291^\circ - 3x &= 180^\circ\\ 3x &= 111^\circ\\ x &= 37^\circ\end{aligned}$$

Lesson 9: Unknown Angle Proofs—Writing Proofs

1. Use the diagram below to prove that $\overline{ST} \perp \overline{QR}$.

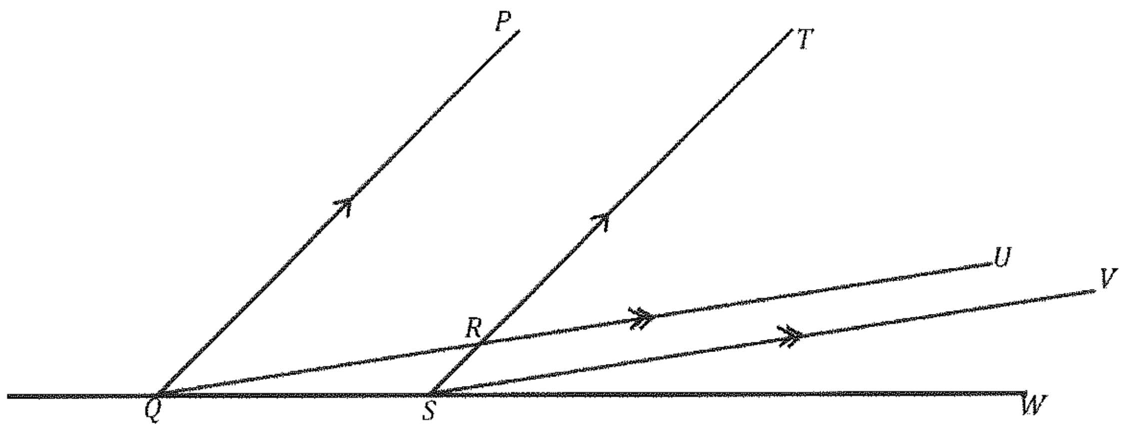
To show that $\overline{ST} \perp \overline{QR}$, I need to first show that $m\angle SVR = 90°$.



$m\angle P + m\angle Q + m\angle PRQ = 180°$	*The sum of the angle measures in a triangle is* $180°$.
$m\angle PRQ = 40°$	*Subtraction property of equality*
$m\angle U + m\angle T + m\angle TSU = 180°$	*The sum of the angle measures in a triangle is* $180°$.
$m\angle TSU = 50°$	*Subtraction property of equality*
$m\angle PRQ + m\angle TSU + m\angle SVR = 180°$	*The sum of the angle measures in a triangle is* $180°$.
$m\angle SVR = 90°$	*Subtraction property of equality*
$\overline{ST} \perp \overline{QR}$	*Perpendicular lines form* $90°$ *angles.*

2. Prove $m\angle PQR = m\angle TSV$.

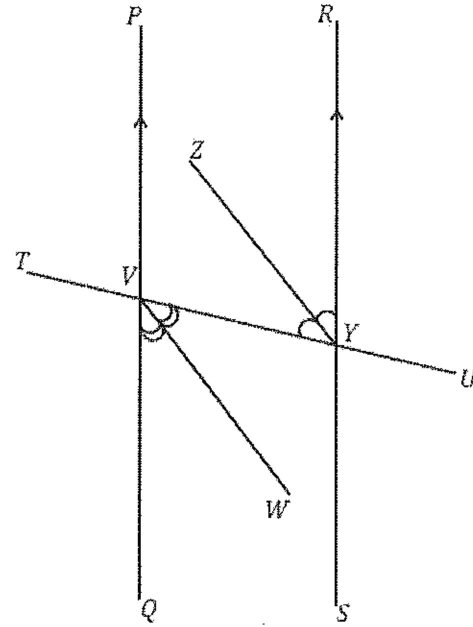
I need to consider how angles $\angle PQR$ and $\angle TSV$ are related to angles I know to be equal in measure in the diagram.



$\overleftrightarrow{PQ} \parallel \overleftrightarrow{TS}$, $\overleftrightarrow{QU} \parallel \overleftrightarrow{SV}$	*Given*
$m\angle PQS = m\angle TSW$, $m\angle RQS = m\angle VSW$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure.*
$m\angle PQR = m\angle PQS - m\angle RQS$, $m\angle TSV = m\angle TSW - m\angle VSW$	*Partition property*
$m\angle PQR = m\angle TSW - m\angle VSW$	*Substitution property of equality*
$m\angle PQR = m\angle TSV$	*Substitution property of equality*

3. In the diagram below, $\overline{VW}$ bisects $\angle QVY$, and $\overline{YZ}$ bisects $\angle VYR$. Prove that $\overline{VW} \parallel \overline{YZ}$.

Since the alternate interior angles along a transversal that cuts parallel lines are equal in measure, the bisected halves are also equal in measure. This will help me determine whether segments VW and YZ are parallel.

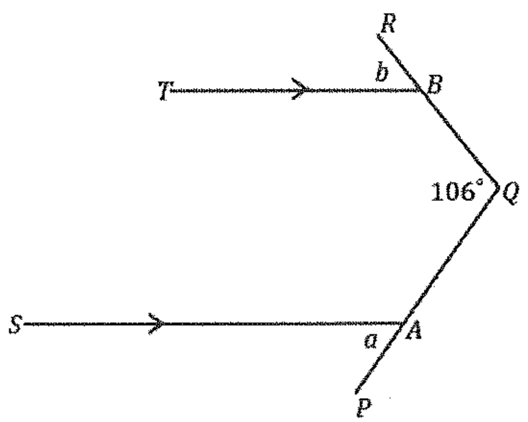


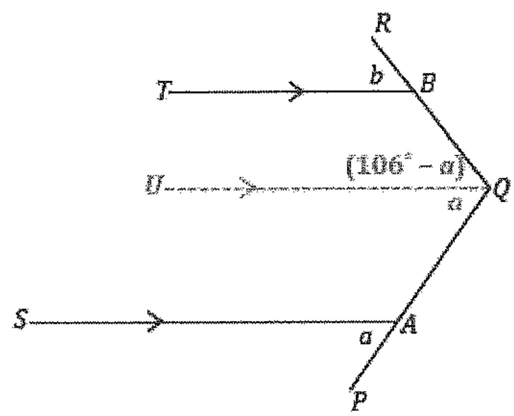
$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$, $\overline{VW}$ *bisects* $\angle QVY$ *and* $\overline{YZ}$ *bisects* $\angle VYR$	*Given*
$m\angle QVY = m\angle VYR$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure.*
$m\angle QVW = m\angle WVY$, $m\angle VYZ = m\angle ZYR$	*Definition of bisect*
$m\angle QVY = m\angle QVW + m\angle WVY$, $m\angle VYR = m\angle VYZ + m\angle ZYR$	*Partition property*
$m\angle QVY = 2(m\angle WVY)$, $m\angle VYR = 2(m\angle VYZ)$	*Substitution property of equality*
$2(m\angle WVY) = 2(m\angle VYZ)$	*Substitution property of equality*
$m\angle WVY = m\angle VYZ$	*Division property of equality*
$\overline{VW} \parallel \overline{YZ}$	*If two lines are cut by a transversal such that a pair of alternate interior angles are equal in measure, then the lines are parallel.*

Lesson 10: Unknown Angle Proofs—Proofs with Constructions

1. Use the diagram below to prove that $b = 106° - a$.

Just as if this were a numeric problem, I need to construct a horizontal line through Q, so I can see the special angle pairs created by parallel lines cut by a transversal.

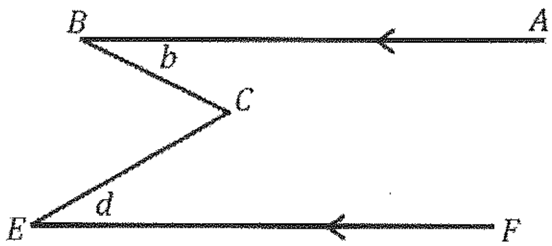


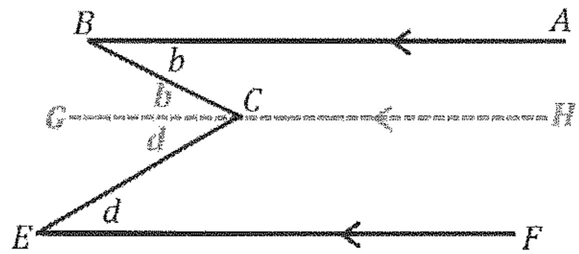


Construct $\overleftrightarrow{UQ}$ *parallel to* $\overleftrightarrow{TB}$ *and* $\overleftrightarrow{SA}$.

$m\angle AQU = a$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure.*
$m\angle UQR = 106° - a$	*Partition property*
$m\angle UQR = b$	*If parallel lines are cut by a transversal, then corresponding angles are equal in measure.*
$b = 106° - a$	*Substitution property of equality*

2. Use the diagram below to prove that $m\angle C = b + d$.



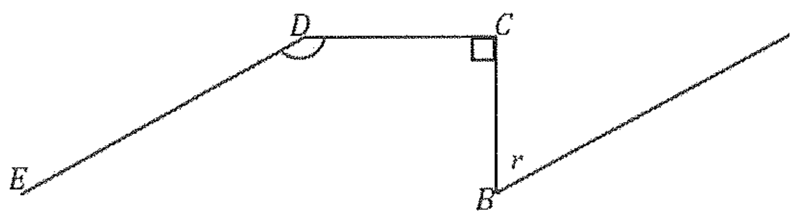


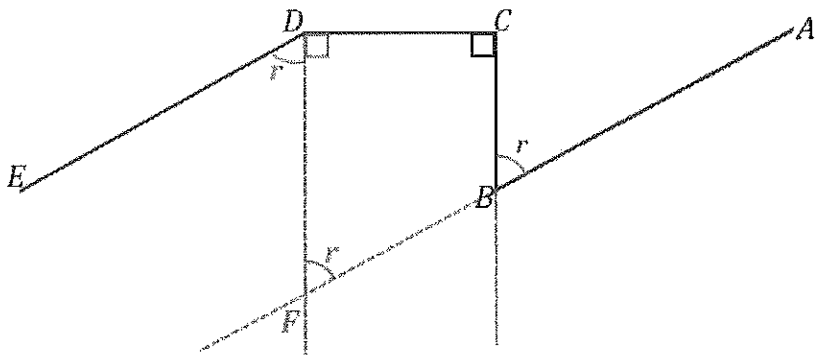
Construct $\overleftrightarrow{GH}$ parallel to $\overleftrightarrow{AB}$ and $\overleftrightarrow{FE}$ through C.

$m\angle BCG = b,\ m\angle ECG = d$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure.*
$m\angle C = b + d$	*Partition property*

3. Use the diagram below to prove that $m\angle CDE = r + 90°$.

> I am going to need multiple constructions to show why the measure of $\angle CDE = r + 90°$.



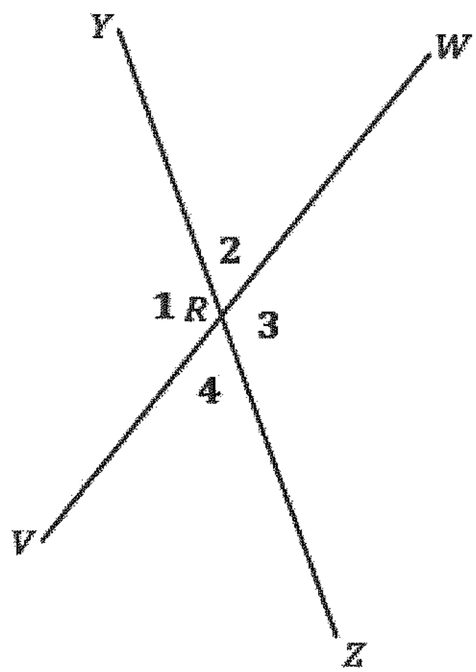


Construct $\overleftrightarrow{DF}$ parallel to $\overleftrightarrow{CB}$. Extend $\overline{AB}$ so that it intersects $\overleftrightarrow{DF}$; extend $\overline{CB}$.

$m\angle CDF + 90° = 180°$	***If parallel lines are cut by a transversal, then same-side interior angles are supplementary.***
$m\angle CDF = 90°$	***Subtraction property of equality***
$m\angle DFB = r$	***If parallel lines are cut by a transversal, then corresponding angles are equal in measure.***
$m\angle EDF = r$	***If parallel lines are cut by a transversal, then alternate interior angles are equal in measure.***
$m\angle CDE = r + 90°$	***Partition property***

Lesson 11: Unknown Angle Proofs—Proofs of Known Facts

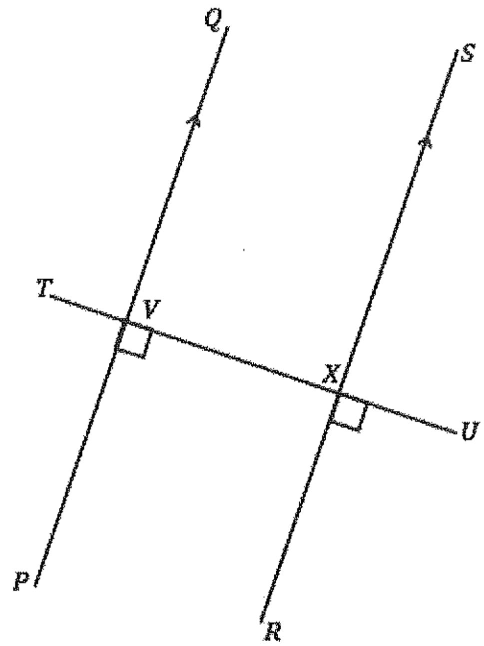
1. **Given:** $\overleftrightarrow{VW}$ and $\overleftrightarrow{YZ}$ intersect at R.
   Prove: $m\angle 2 = m\angle 4$



$\overleftrightarrow{VW}$ and $\overleftrightarrow{YZ}$ intersect at R.	*Given*
$m\angle 1 + m\angle 2 = 180°$; $m\angle 1 + m\angle 4 = 180°$	*Angles on a line sum to $180°$.*
$m\angle 1 + m\angle 2 = m\angle 1 + m\angle 4$	*Substitution property of equality*
$m\angle 2 = m\angle 4$	*Subtraction property of equality*

2. **Given:** $\overleftrightarrow{PQ} \perp \overleftrightarrow{TU}; \overleftrightarrow{RS} \perp \overleftrightarrow{TU}$

   Prove: $\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$

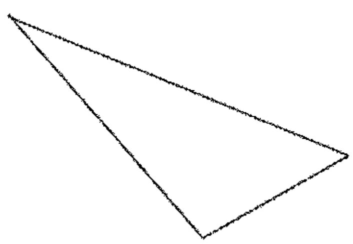
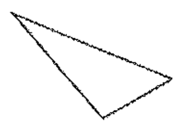
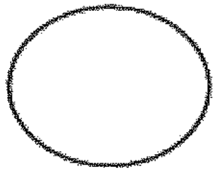
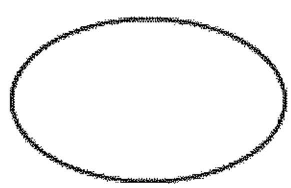


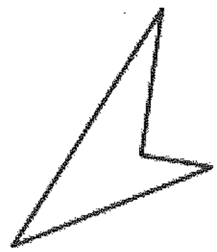
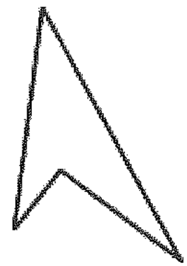
$\overleftrightarrow{PQ} \perp \overleftrightarrow{TU}; \overleftrightarrow{RS} \perp \overleftrightarrow{TU}$	*Given*
$m\angle QVX = 90°; m\angle SXU = 90°$	*Perpendicular lines form 90° angles.*
$m\angle QVX = m\angle SXU$	*Transitive property of equality*
$\overleftrightarrow{PQ} \parallel \overleftrightarrow{RS}$	*If two lines are cut by a transversal such that a pair of corresponding angles are equal in measure, then the lines are parallel.*

Lesson 12: Transformations—The Next Level

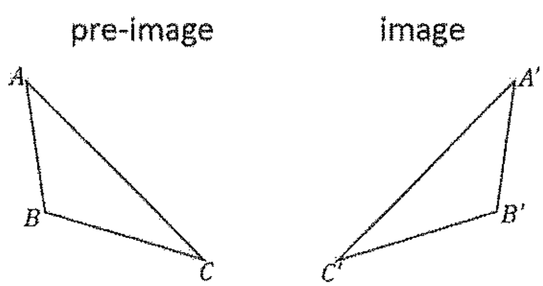
1. Recall that a transformation F of the plane is a function that assigns to each point P of the plane a unique point $F(P)$ in the plane. Of the countless kinds of transformations, a subset exists that preserves lengths and angle measures. In other words, they are transformations that do not distort the figure. These transformations, specifically reflections, rotations, and translations, are called basic rigid motions.

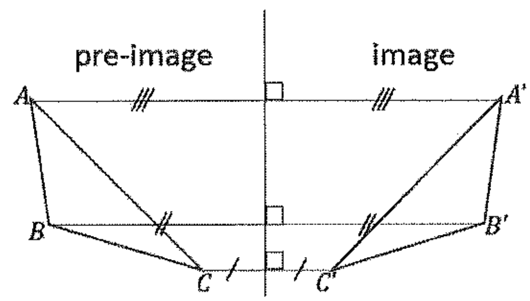
   Examine each pre-image and image pair. Determine which pairs demonstrate a rigid motion applied to the pre-image.

	Pre-Image	**Image**	**Is this transformation an example of a rigid motion? Explain.**
a.			*No, this transformation did not preserve lengths, even though it seems to have preserved angle measures.*
b.			*Yes, this is a rigid motion—a translation.*
c.			*Yes, this is a rigid motion—a reflection.*
d.			*No, this transformation did not preserve lengths or angle measures.*

e.			***Yes, this is a rigid motion—a rotation.***

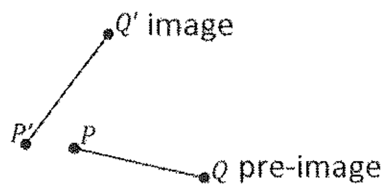
2. Each of the following pairs of diagrams shows the same figure as a pre-image and as a post-transformation image. Each of the second diagrams shows the details of how the transformation is performed. Describe what you see in each of the second diagrams.

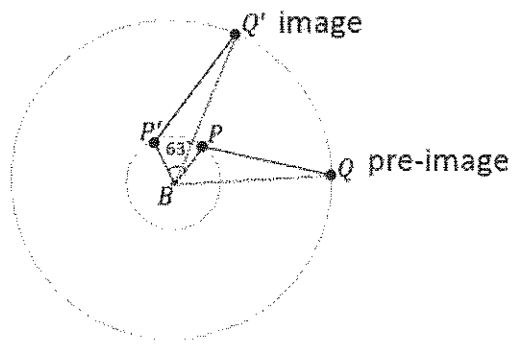




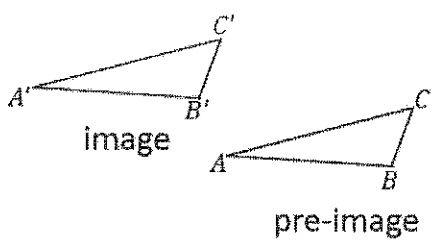
The line that the pre-image is reflected over is the perpendicular bisector of each of the segments joining the corresponding vertices of the triangles.

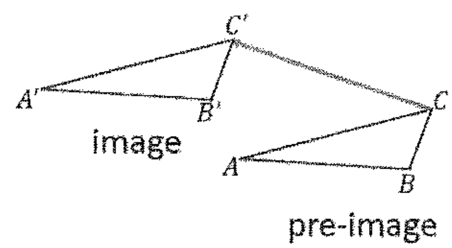
For each of the transformations, I must describe all the details that describe the "mechanics" of how the transformation works. For example, I see that there are congruency marks on each half of the segments that join the corresponding vertices and that each segment is perpendicular to the line of reflection. This is essential to how the reflection works.





The segment PQ is rotated counterclockwise about B by $63°$. The path that describes how P maps to P' is a circle with center B and radius $\overline{BP}$; P moves counterclockwise along the circle. $\angle PBP'$ has a measure of $63°$. A similar case can be made for Q.





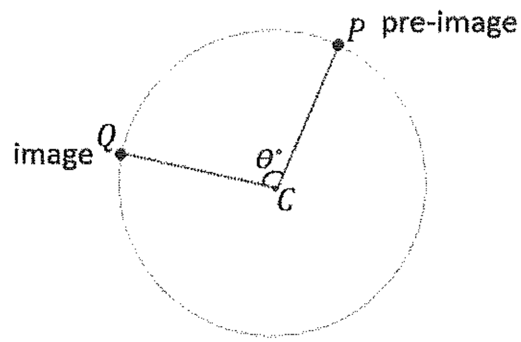
The pre-image $\triangle ABC$ has been translated the length and direction of vector $\overrightarrow{CC'}$.

Lesson 13: Rotations

1. Recall the definition of rotation:

   For $0° < \theta° < 180°$, the rotation of θ degrees around the center C is the transformation $R_{C,\theta}$ of the plane defined as follows:

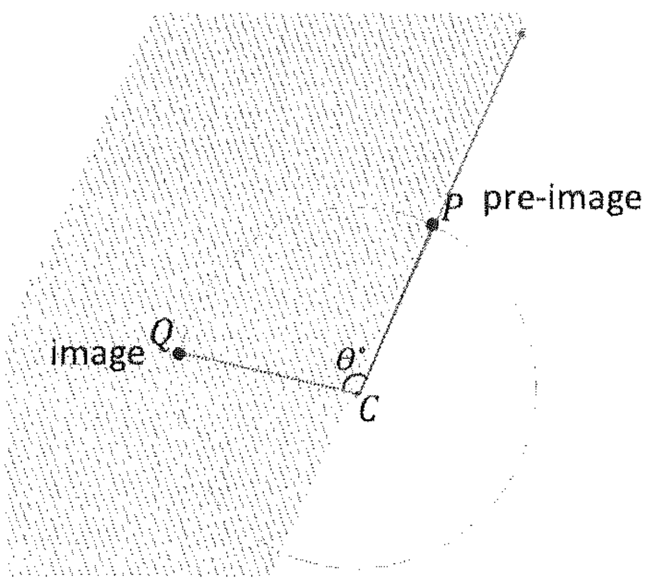
   1. For the center point C, $R_{C,\theta}(C) = C$, and
   2. For any other point P, $R_{C,\theta}(P)$ is the point Q that lies in the counterclockwise half-plane of $\overrightarrow{CP}$, such that $CQ = CP$ and $m\angle PCQ = \theta°$.



   a. Which point does the center C map to once the rotation has been applied?

   By the definition, the center C maps to itself: $R_{C,\theta}(C) = C$.

   b. The image of a point P that undergoes a rotation $R_{C,\theta}$ is the image point Q: $R_{C,\theta}(P) = Q$. Point Q is said to lie in the counterclockwise half plane of $\overrightarrow{CP}$. Shade the counterclockwise half plane of $\overrightarrow{CP}$.



I must remember that a half plane is a line in a plane that separates the plane into two sets.

c. Why does part (2) of the definition include $CQ = CP$? What relationship does $CQ = CP$ have with the circle in the diagram above?

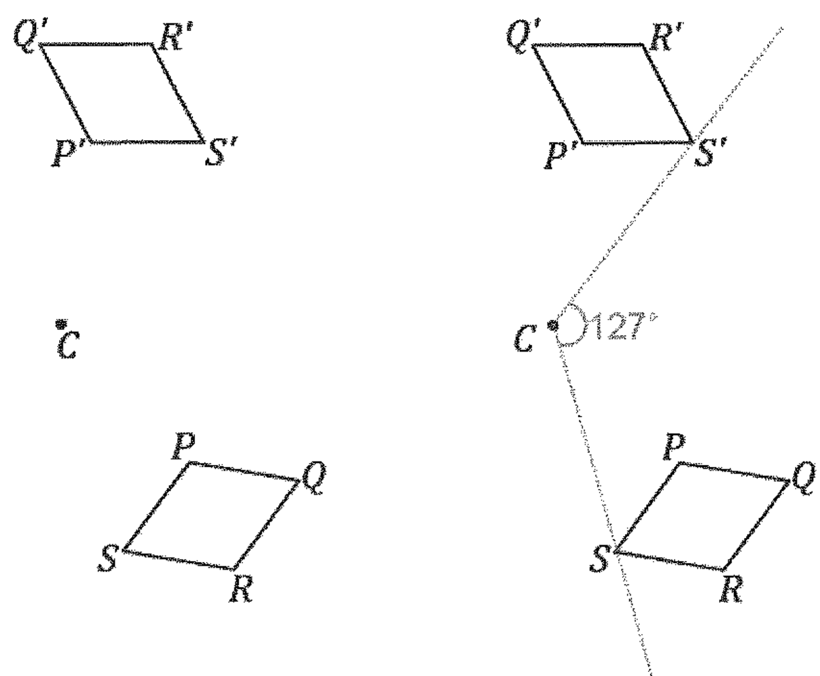
$CQ = CP$ describes how P maps to Q. The rotation, a function, describes a path such that P "rotates" (let us remember there is no actual motion) along the circle C with radius CP (and thereby also of radius CQ).

d. Based on the figure on the prior page, what is the angle of rotation, and what is the measure of the angle of rotation?

The angle of rotation is $\angle PCQ$, and the measure is $\theta°$.

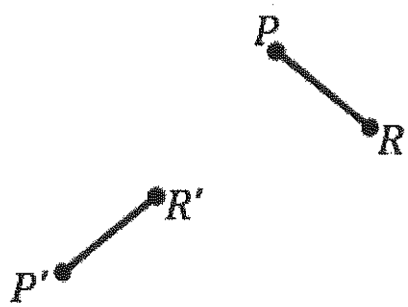
2. Use a protractor to determine the angle of rotation.

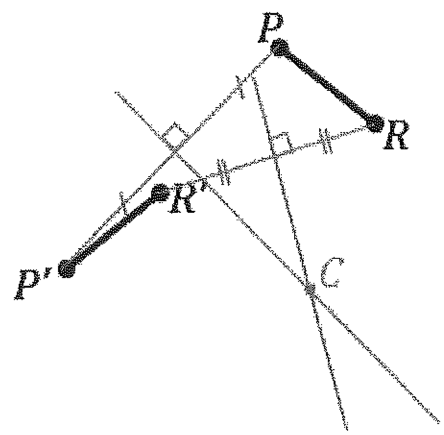
I must remember that the angle of rotation is found by forming an angle from any pair of corresponding points and the center of rotation; the measure of this angle is the angle of rotation.



3. Determine the center of rotation for the following pre-image and image.

I must remember that the center of rotation is located by the following steps: (1) join two pairs of corresponding points in the pre-image and image, (2) take the perpendicular bisector of each segment, and finally (3) identify the intersection of the bisectors as the center of rotation.



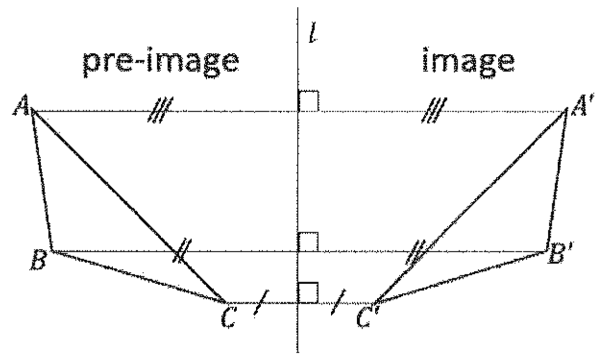


Lesson 14: Reflections

1. Recall the definition of reflection:

   For a line l in the plane, a reflection across l is the transformation r_l of the plane defined as follows:

   1. For any point P on the line l, $r_l(P) = P$, and
   2. For any point P not on l, $r_l(P)$ is the point Q so that l is the perpendicular bisector of the segment PQ.



   a. Where do the points that belong to a line of reflection map to once the reflection is applied?

   Any point P on the line of reflection maps to itself: $r_l(P) = P$.

   I can model a reflection by folding paper: The fold itself is the line of reflection.

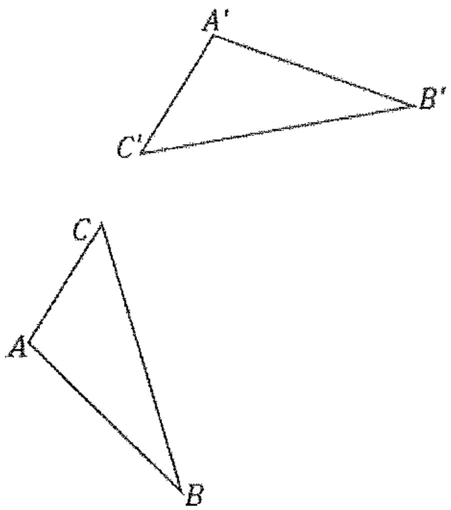
   b. Once a reflection is applied, what is the relationship between a point, its reflected image, and the line of reflection? For example, based on the diagram above, what is the relationship between A, A', and line l?

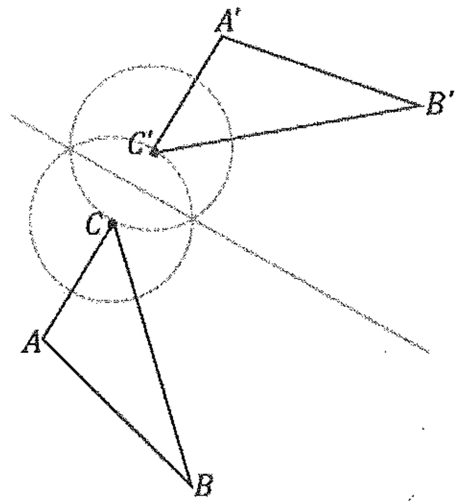
   Line l is the perpendicular bisector to the segment that joins A and A'.

   c. Based on the diagram above, is there a relationship between the distance from B to l and from B' to l?

   Any pair of corresponding points is equidistant from the line of reflection.

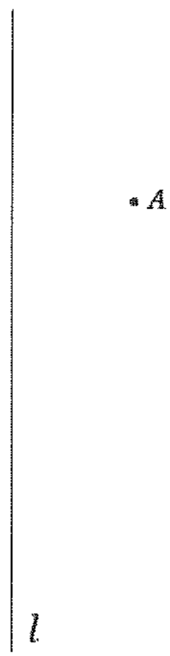
2. Using a compass and straightedge, determine the line of reflection for pre-image $\triangle ABC$ and $\triangle A'B'C'$. Write the steps to the construction.

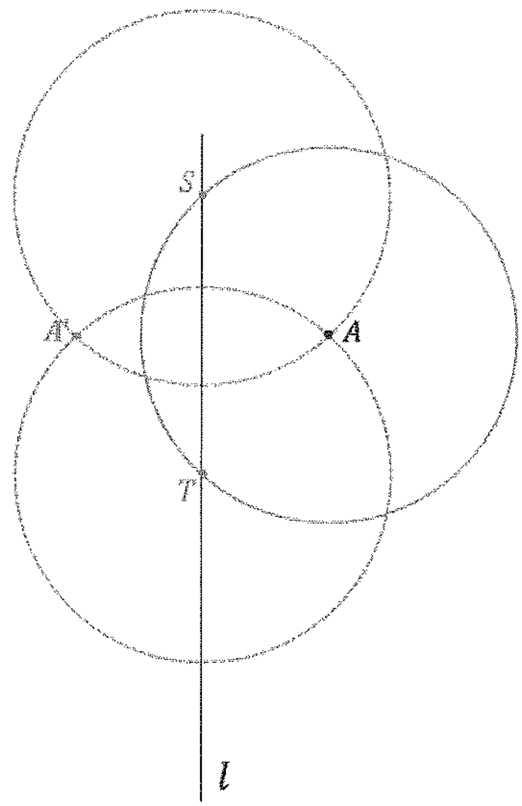




1. ***Draw circle C: center C, radius CC'.***
2. ***Draw circle C': center C', radius $C'C$.***
3. ***Draw a line through the points of intersection between circles C and C'.***

3. Using a compass and straightedge, reflect point A over line l. Write the steps to the construction.





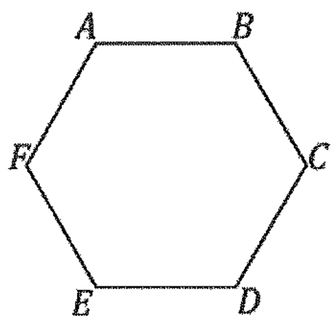
1. ***Draw circle A such that the circle intersects with line l in two locations, S and T.***
2. ***Draw circle S: center S, radius SA.***
3. ***Draw circle T: center T, radius TA.***
4. ***Label the intersection of circles S and T as A'.***

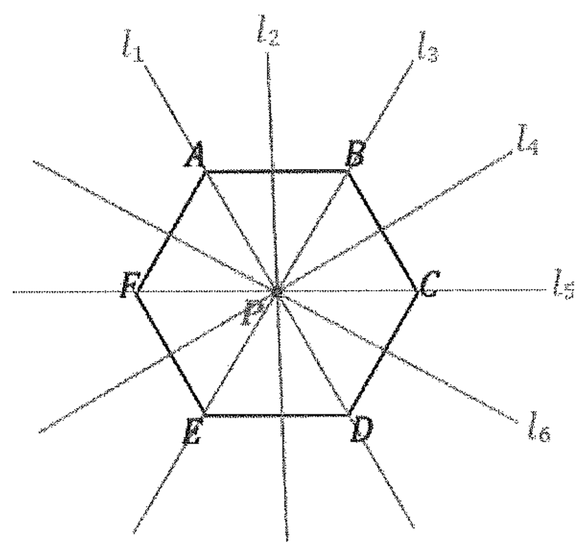
EUREKA MATH

Lesson 15: Rotations, Reflections, and Symmetry

1. A symmetry of a figure is a basic rigid motion that maps the figure back onto itself. A figure is said to have line symmetry if there exists a line (or lines) so that the image of the figure when reflected over the line(s) is itself. A figure is said to have nontrivial rotational symmetry if a rotation of greater than 0° but less than 360° maps a figure back to itself. A trivial symmetry is a transformation that maps each point of a figure back to the same point (i.e., in terms of a function, this would be $f(x) = x$). An example of this is a rotation of 360°.

   a. Draw all lines of symmetry for the equilateral hexagon below. Locate the center of rotational symmetry.





   b. How many of the symmetries are rotations (of an angle of rotation less than or equal to 360°)? What are the angles of rotation that yield symmetries?

   6, including the identity symmetry. The angles of rotation are: $60°$, $120°$, $180°$, $240°$, $300°$, *and* $360°$.

   c. How many of the symmetries are reflections?

   6

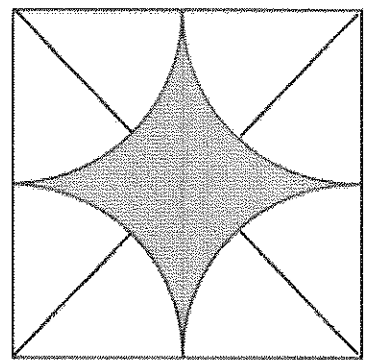
d. How many places can vertex A be moved by some symmetry of the hexagon?

A can be moved to 6 places—A, B, C, D, E, and F.

e. For a given symmetry, if you know the image of A, how many possibilities exist for the image of B?

2

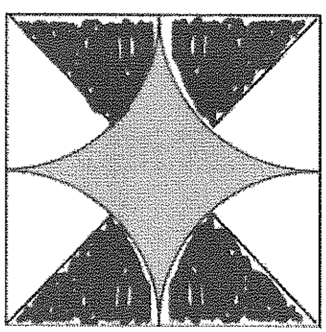
2. Shade as few of the nine smaller sections as possible so that the resulting figure has



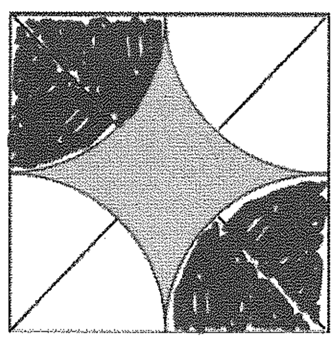
a. Only one vertical and one horizontal line of symmetry.
b. Only two lines of symmetry about the diagonals.
c. Only one horizontal line of symmetry.
d. Only one line of symmetry about a diagonal.
e. No line of symmetry.

Possible solutions:

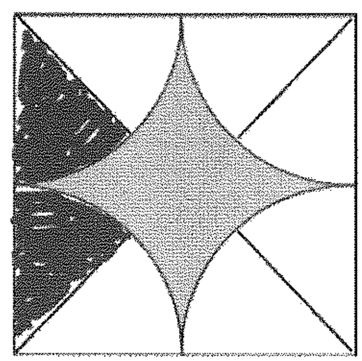
a.



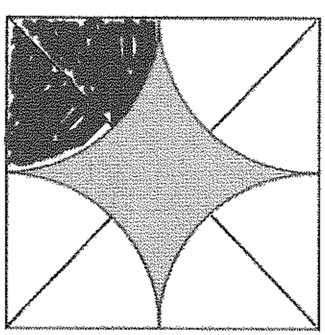
b.



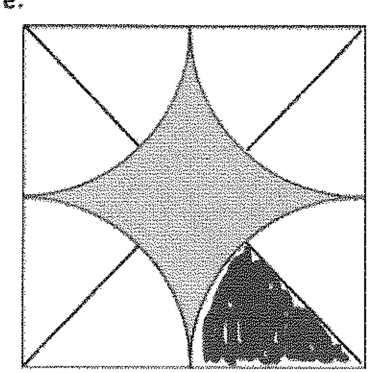
c.



d.



e.

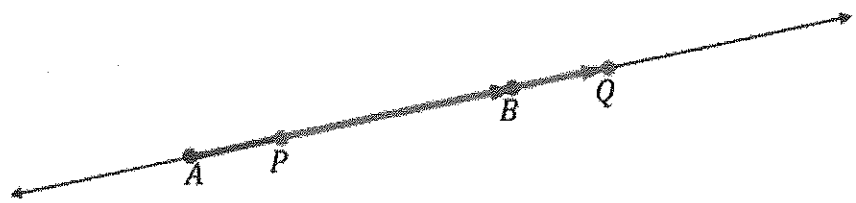


Lesson 16: Translations

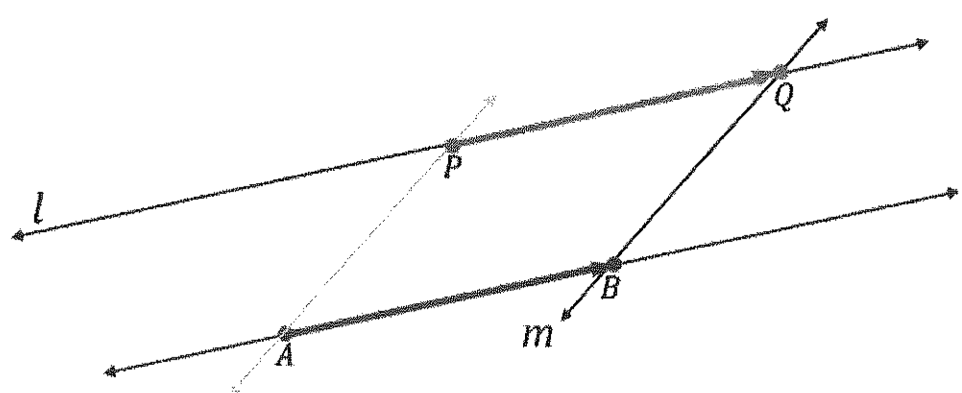
1. Recall the definition of translation:

   For vector $\overrightarrow{AB}$, the translation along $\overrightarrow{AB}$ is the transformation $T_{\overrightarrow{AB}}$ of the plane defined as follows:

   1. For any point P on $\overleftrightarrow{AB}$, $T_{\overrightarrow{AB}}(P)$ is the point Q on $\overleftrightarrow{AB}$ so that $\overrightarrow{PQ}$ has the same length and the same direction as $\overrightarrow{AB}$, and

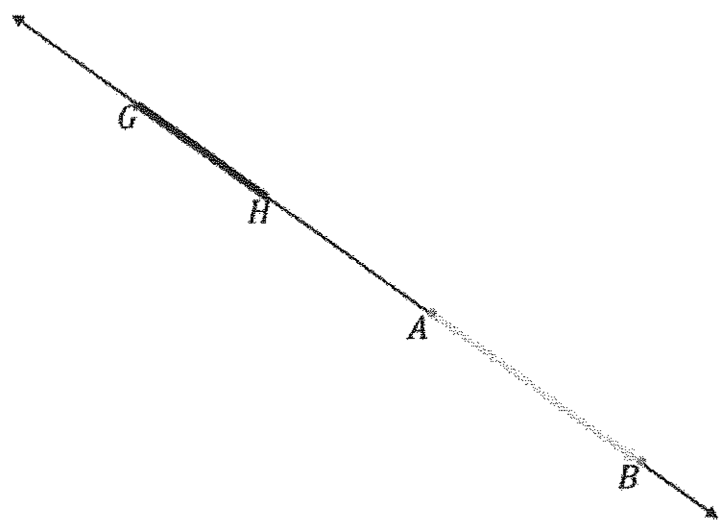


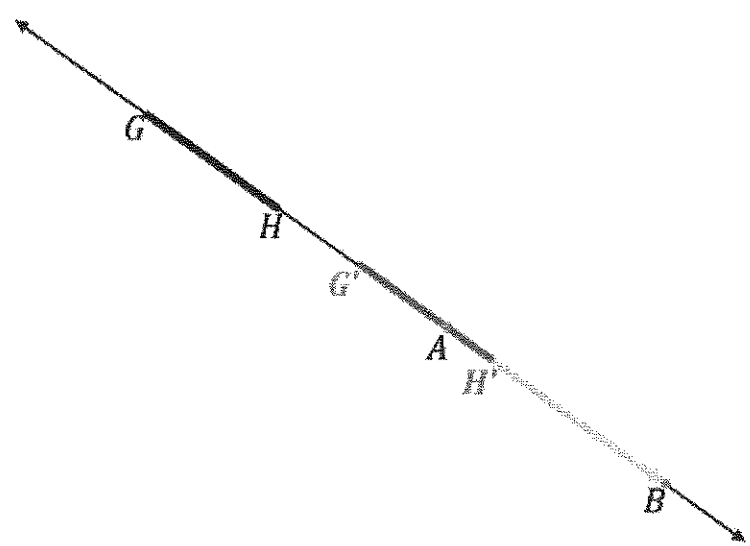
   2. For any point P not on $\overleftrightarrow{AB}$, $T_{\overrightarrow{AB}}(P)$ is the point Q obtained as follows. Let l be the line passing through P and parallel to $\overleftrightarrow{AB}$. Let m be the line passing through B and parallel to $\overleftrightarrow{AP}$. The point Q is the intersection of l and m.



2. Use a compass and straightedge to translate segment $\overline{GH}$ along vector $\overrightarrow{AB}$.

To find G', I must mark off the length of $\overrightarrow{AB}$ in the direction of the vector from G. I will repeat these steps to locate H'.





EUREKA MATH

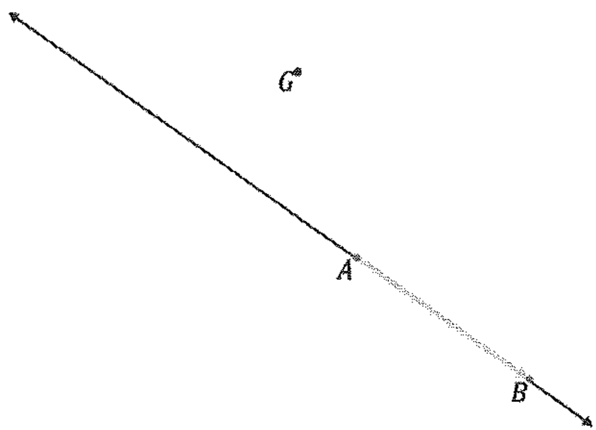
3. Use a compass and straightedge to translate point G along vector $\overrightarrow{AB}$. Write the steps to this construction.

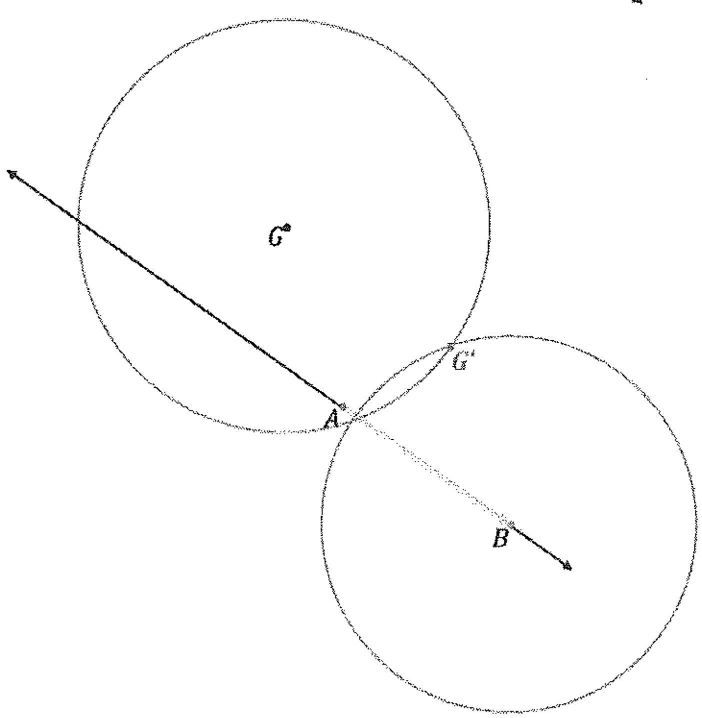
   1. Draw circle G: center G, radius AB.

   2. Draw circle B: center B, radius AG.

   3. Label the intersection of circle G and circle B as G'. (Circles G and B intersect in two locations; pick the intersection so that A and G' are in opposite half planes of $\overleftrightarrow{BG}$.)

To find G', my construction is really resulting in locating the fourth vertex of a parallelogram.





Lesson 17: Characterize Points on a Perpendicular Bisector

1. Perpendicular bisectors are essential to the rigid motions reflections and rotations.

   a. How are perpendicular bisectors essential to reflections?

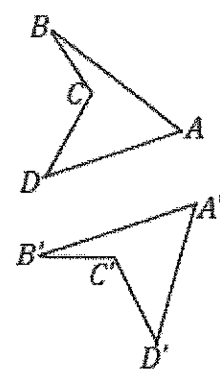
   I can re-examine perpendicular bisectors (in regard to reflections) in Lesson 14 and rotations in Lesson 13.

   The line of reflection is a perpendicular bisector to the segment that joins each pair of pre-image and image points of a reflected figure.

   b. How are perpendicular bisectors essential to rotations?

   Perpendicular bisectors are key to determining the center of a rotation. The center of a rotation is determined by joining two pairs of pre-image and image points and constructing the perpendicular bisector of each of the segments. Where the perpendicular bisectors intersect is the center of the rotation.

2. Rigid motions preserve distance, or in other words, the image of a figure that has had a rigid motion applied to it will maintain the same lengths as the original figure.



   a. Based on the following rotation, which of the following statements must be true?

   i. $AD = A'D'$ ***True***

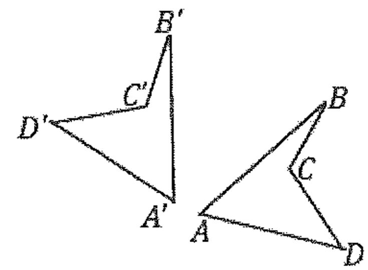
   ii. $BB' = CC'$ ***False***

   iii. $AC = A'C'$ ***True***

   iv. $BD = B'D'$ ***True***

   v. $CD' = C'D$ ***False***

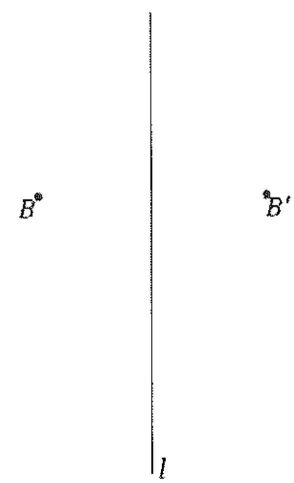
b. Based on the following rotation, which of the following statements must be true?



i. $CC' = BB'$ *False*

ii. $BC = B'C'$ *True*

3. In the following figure, point B is reflected across line l.



c. What is the relationship between B, B', and l?

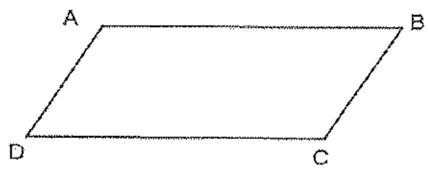
Line l is the perpendicular bisector of $\overline{BB'}$.

d. What is the relationship between B, B', and any point P on l?

B and B' are equidistant from line l and therefore equidistant from any point P on l.

Lesson 18: Looking More Carefully at Parallel Lines

1. Given that $\angle B$ and $\angle C$ are supplementary and $\overline{AD} \parallel \overline{BC}$, prove that $m\angle A = m\angle C$.



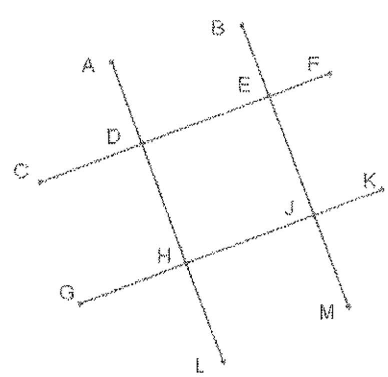
$\angle B$ and $\angle C$ are supplementary.	*Given*
$\overline{AD} \parallel \overline{BC}$	*Given*
$\angle B$ and $\angle A$ are supplementary.	*If two parallel lines are cut by a transversal, then same-side interior angles are supplementary.*
$m\angle B + m\angle A = 180°$	*Definition of supplementary angles*
$m\angle B + m\angle C = 180°$	*Definition of supplementary angles*
$m\angle B + m\angle A = m\angle B + m\angle C$	*Substitution property of equality*
$m\angle A = m\angle C$	*Subtraction property of equality*

If $\overline{AD} \parallel \overline{BC}$, then $\angle A$ and $\angle B$ are supplementary because they are same-side interior angles.

2. Mathematicians state that *if a transversal is perpendicular to two distinct lines, then the distinct lines are parallel.* Prove this statement. (Include a labeled drawing with your proof.)

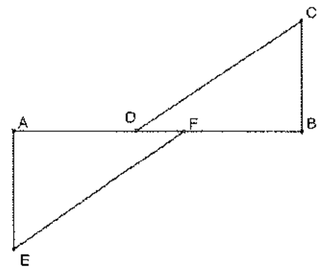
$\overline{AL} \perp \overline{CF}$, $\overline{AL} \perp \overline{GK}$	*Given*
$m\angle ADC = 90°$	*Definition of perpendicular lines*
$m\angle AHG = 90°$	*Definition of perpendicular lines*
$m\angle ADC = m\angle AHG$	*Substitution property of equality*
$\overline{CF} \parallel \overline{GK}$	*If a transversal cuts two lines such that corresponding angles are equal in measure, then the two lines are parallel.*

If a transversal is perpendicular to one of the two lines, then it meets that line at an angle of 90°. Since the lines are parallel, I can use corresponding angles of parallel lines to show that the transversal meets the other line, also at an angle of 90°.



EUREKA MATH

3. In the figure, D and F lie on $\overline{AB}$, $m\angle BDC = m\angle AFE$, and $m\angle C = m\angle E$. Prove that $\overline{AE} \parallel \overline{CB}$.



I know that in any triangle, the three angle measures sum to $180°$. If two angles in one triangle are equal in measure to two angles in another triangle, then the third angles in each triangle must be equal in measure.

$m\angle BDC = m\angle AFE$	*Given*
$m\angle C = m\angle E$	*Given*
$m\angle BDC + m\angle C + m\angle B = 180°$	*Sum of the angle measures in a triangle is $180°$.*
$m\angle AFE + m\angle E + m\angle A = 180°$	*Sum of the angle measures in a triangle is $180°$.*
$m\angle AFE + m\angle E + m\angle B = 180°$	*Substitution property of equality*
$m\angle B = 180° - m\angle AFE - m\angle E$	*Subtraction property of equality*
$m\angle A = 180° - m\angle AFE - m\angle E$	*Subtraction property of equality*
$m\angle A = m\angle B$	*Substitution property of equality*
$\overline{AE} \parallel \overline{CB}$	*If two lines are cut by a transversal such that a pair of alternate interior angles are equal in measure, then the lines are parallel.*

Lesson 19: Construct and Apply a Sequence of Rigid Motions

1. Use your understanding of congruence to answer each of the following.

   a. Why can't a square be congruent to a regular hexagon?

   A square cannot be congruent to a regular hexagon because there is no rigid motion that takes a figure with four vertices to a figure with six vertices.

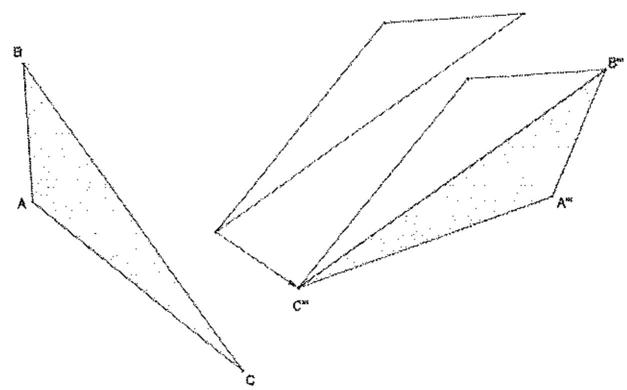
   To be congruent, the figures must have a correspondence of vertices. I know that a square has four vertices, and a regular hexagon has six vertices. No matter what sequence of rigid motions I use, I cannot correspond all vertices of the hexagon with vertices of the square.

   b. Can a square be congruent to a rectangle?

   A square can only be congruent to a rectangle if the sides of the rectangle are all the same length as the sides of the square. This would mean that the rectangle is actually a square.

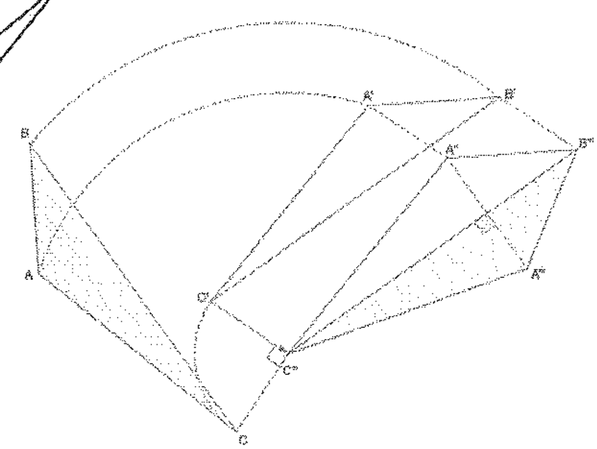
   I know that by definition, a rectangle is a quadrilateral with four right angles.

2. The series of figures shown in the diagram shows the images of $\triangle ABC$ under a sequence of rigid motions in the plane. Use a piece of patty paper to find and describe the sequence of rigid motions that shows $\triangle ABC \cong \triangle A'''B'''C'''$. Label the corresponding image points in the diagram using prime notation.



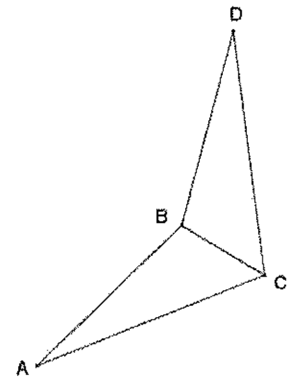
First, a rotation of $90°$ about point C''' in a clockwise direction takes $\triangle ABC$ to $\triangle A'B'C'$. Next, a translation along $\overrightarrow{C'C'''}$ takes $\triangle A'B'C'$ to $\triangle A''B''C''$. Finally, a reflection over $\overline{B''C''}$ takes $\triangle A''B''C''$ to $\triangle A'''B'''C'''$.

I can see that $\triangle A'''B'''C'''$ is turned on the plane compared to $\triangle ABC$, so I should look for a rotation in my sequence. I also see a vector, so there might be a translation, too.



EUREKA MATH™

3. In the diagram to the right, $\triangle ABC \cong \triangle DBC$.



   a. Describe two distinct rigid motions, or sequences of rigid motions, that map A onto D.

      The most basic of rigid motions mapping A onto D is a reflection over $\overline{BC}$.

      Another possible sequence of rigid motions includes a rotation about B of degree measure equal to $m\angle ABD$ followed by a reflection over $\overline{BD}$.

   In the given congruence statement, the vertices of the first triangle are named in a clockwise direction, but the corresponding vertices of the second triangle are named in a counterclockwise direction. The change in orientation tells me that a reflection must be involved.

   b. Using the congruence that you described in your response to part (a), what does $\overline{AC}$ map to?

      By a reflection of the plane over $\overline{BC}$, $\overline{AC}$ maps to $\overline{DC}$.

   c. Using the congruence that you described in your response to part (a), what does $\overline{BC}$ map to?

      By a reflection of the plane over $\overline{BC}$, $\overline{BC}$ maps to itself because it lies in the line of reflection.

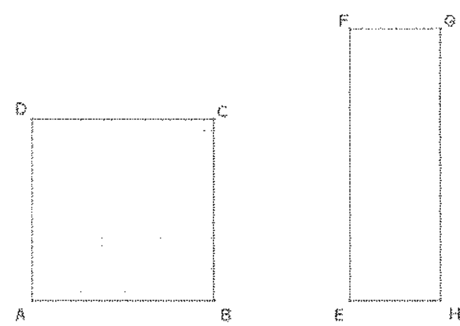
Lesson 20: Applications of Congruence in Terms of Rigid Motions

1. Give an example of two different quadrilaterals and a correspondence between their vertices such that (a) all four corresponding angles are congruent, and (b) none of the corresponding sides are congruent.

   I know that some quadrilaterals have matching angle characteristics such as squares and rectangles. Both of these quadrilaterals are required to have four right angles.

   The following represents one of many possible answers to this problem.

   Square $ABCD$ and rectangle $EFGH$ meet the above criteria. By definition, both quadrilaterals are required to have four right angles, which means that any correspondence of vertices will map together congruent angles. A square is further required to have all sides of equal length, so as long as none of the sides of the rectangle are equal in length to the sides of the square, the criteria are satisfied.



2. Is it possible to give an example of two triangles and a correspondence between their vertices such that only two of the corresponding angles are congruent? Explain your answer.

   I know that every triangle, no matter what size or classification, has an angle sum of $180°$.

   Any triangle has three angles, and the sum of the measures of those angles is $180°$. If two triangles are given such that one pair of angles measure $x°$ and a second pair of angles measure $y°$, then by the angle sum of a triangle, the remaining angle would have to have a measure of $(180 - x - y)°$. This means that the third pair of corresponding angles must also be congruent, so no, it is not possible.

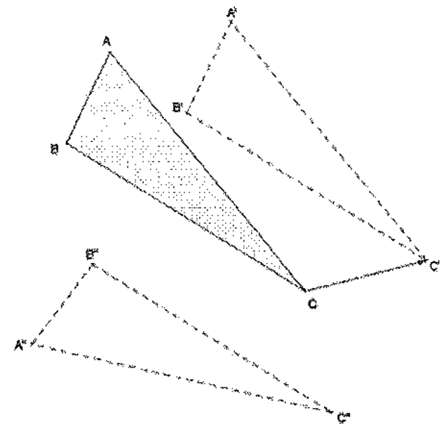
3. Translations, reflections, and rotations are referred to as *rigid motions*. Explain why the term *rigid* is used.

   The term *rigid* means not flexible.

   Each of the rigid motions is a transformation of the plane that can be modelled by tracing a figure on the plane onto a transparency and transforming the transparency by following the given function rule. In each case, the image is identical to the pre-image because translations, rotations, and reflections preserve distance between points and preserve angles between lines. The transparency models rigidity.

Lesson 21: Correspondence and Transformations

1. The diagram below shows a sequence of rigid motions that maps a pre-image onto a final image.
   a. Identify each rigid motion in the sequence, writing the composition using function notation.

   The first rigid motion is a translation along $\overrightarrow{CC'}$ to yield triangle $A'B'C'$. The second rigid motion is a reflection over $\overline{BC}$ to yield triangle $A''B''C''$. In function notation, the sequence of rigid motions is $r_{\overline{BC}}\left(T_{\overrightarrow{CC'}}(\triangle ABC)\right)$.

   b. Trace the congruence of each set of corresponding sides and angles through all steps in the sequence, proving that the pre-image is congruent to the final image by showing that every side and every angle in the pre-image maps onto its corresponding side and angle in the image.

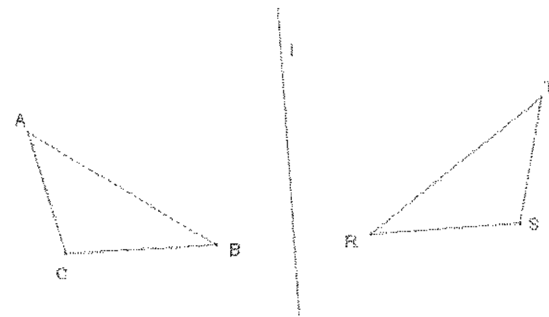
   Sequence of corresponding sides: $\overline{AB} \rightarrow \overline{A''B''}$, $\overline{BC} \rightarrow \overline{B''C''}$, and $\overline{AC} \rightarrow \overline{A''C''}$.

   Sequence of corresponding angles: $\angle A \rightarrow \angle A''$, $\angle B \rightarrow \angle B''$, and $\angle C \rightarrow \angle C''$.

   c. Make a statement about the congruence of the pre-image and the final image.

   $\triangle ABC \cong \triangle A''B''C''$

2. Triangle TRS is a reflected image of triangle ABC over a line ℓ. Is it possible for a translation or a rotation to map triangle TRS back to the corresponding vertices in its pre-image, triangle ABC? Explain why or why not.



When I look at the words printed on my t-shirt in a mirror, the order of the letters, and even the letters themselves, are completely backward. I can see the words correctly if I look at a reflection of my reflection in another mirror.

The orientation of three non-collinear points will change under a reflection of the plane over a line. This means that if you consider the correspondence $A \to T$, $B \to R$, and $C \to S$, if the vertices A, B, and C are oriented in a clockwise direction on the plane, then the vertices T, R, and S will be oriented in a counterclockwise direction. It is possible to map T to A, S to C, or R to B individually under a variety of translations or rotations; however, a reflection is required in order to map each of T, R, and S to its corresponding pre-image.

3. Describe each transformation given by the sequence of rigid motions below, in function notation, using the correct sequential order.

$$r_m\left(T_{\overrightarrow{AB}}\left(R_{X,60^\circ}(\triangle XYZ)\right)\right)$$

I know that in function notation, the innermost function, in this case $R_{X,60^\circ}(\triangle XYZ)$, is the first to be carried out on the points in the plane.

The first rigid motion is a rotation of $\triangle XYZ$ around point X of 60°. Next is a translation along $\overrightarrow{AB}$. The final rigid motion is a reflection over a given line m.

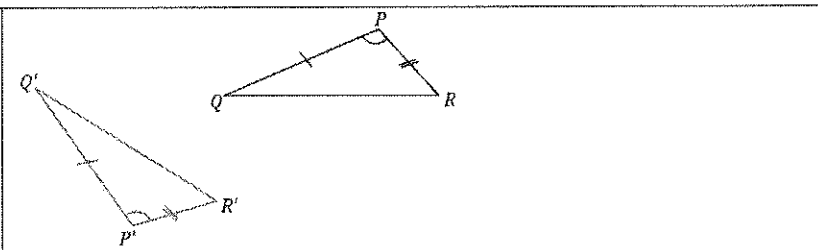
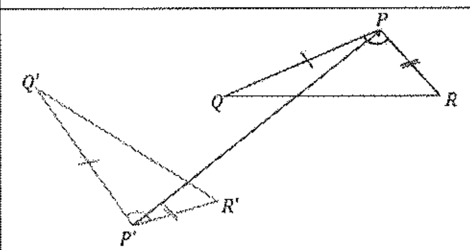
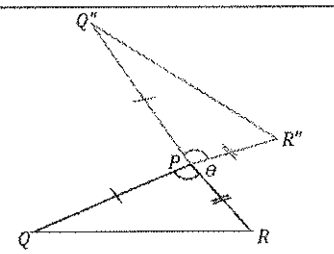
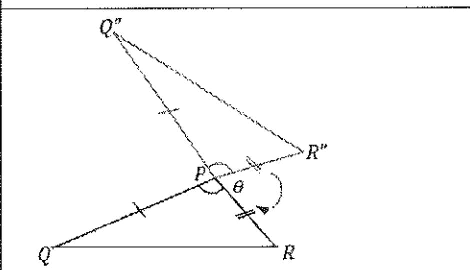
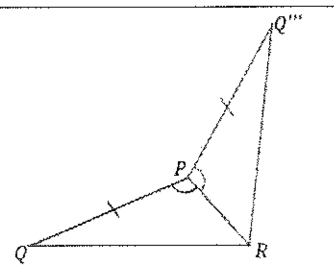
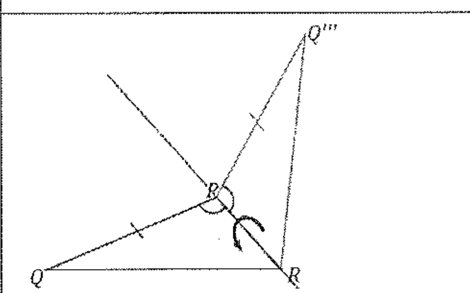
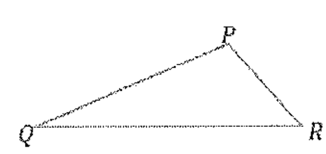
Lesson 22: Congruence Criteria for Triangles—SAS

1. We define two figures as congruent if there exists a finite composition of rigid motions that maps one onto the other. The following triangles meet the Side-Angle-Side criterion for congruence. The criterion tells us that only a few parts of two triangles, as well as a correspondence between them, is necessary to determine that the two triangles are congruent.

   Describe the rigid motion in each step of the proof for the SAS criterion:

   Given: $\triangle PQR$ and $\triangle P'Q'R'$ so that $PQ = P'Q'$ (Side), $m\angle P = m\angle P'$ (Angle), and $PR = P'R'$ (Side).

   Prove: $\triangle PQR \cong \triangle P'Q'R'$

<table>
<tr><td>1</td><td colspan="2">
</td><td>Given, distinct triangles $\triangle PQR$ and $\triangle P'Q'R'$ so that $PQ = P'Q'$, $m\angle P = m\angle P'$, and $PR = P'R'$.</td></tr>
<tr><td>2</td><td>
</td><td>
</td><td>$T_{\overrightarrow{P'P}}(\triangle P'Q'R') = \triangle PQ''R''$; $\triangle P'Q'R'$ is translated along vector $\overrightarrow{P'P}$. $\triangle PQR$ and $\triangle PQ''R''$ share common vertex P.</td></tr>
<tr><td>3</td><td>
</td><td>
</td><td>$R_{P,-\theta}(\triangle PQ''R'') = \triangle PQ'''R$; $\triangle PQ''R''$ is rotated about center P by $\theta°$ clockwise. $\triangle PQR$ and $\triangle PQ'''R$ share common side $\overline{PR}$.</td></tr>
<tr><td>4</td><td>
</td><td>
</td><td>$r_{\overline{PR}}(\triangle PQ'''R) = \triangle PQR$; $\triangle PQ'''R$ is reflected across $\overleftrightarrow{PR}$ $\triangle PQ'''R$ coincides with $\triangle PQR$.</td></tr>
</table>

a. In Step 3, how can we be certain that R'' will map to R?

By assumption, $PR'' = PR$. This means that not only will $\overrightarrow{PR''}$ map to $\overrightarrow{PR}$ under the rotation, but R'' will map to R.

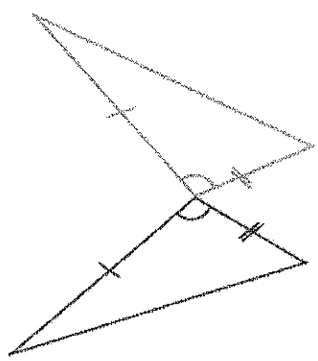
> I must remember that if the lengths of PR'' and PR were not known, the rotation would result in coinciding rays $\overrightarrow{PR''}$ and $\overrightarrow{PR}$ but nothing further.

b. In Step 4, how can we be certain that Q''' will map to Q?

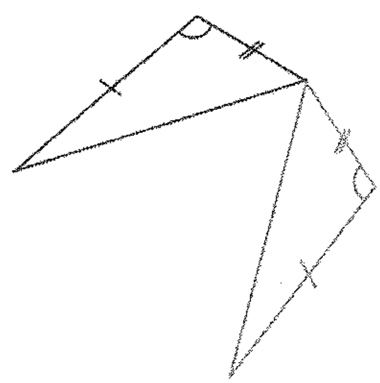
Rigid motions preserve angle measures. This means that $m\angle QPR = m\angle Q'''PR$. Then the reflection maps $\overrightarrow{PQ'''}$ to $\overrightarrow{PQ}$. Since $PQ''' = PQ$, Q''' will map to Q.

c. In this example, we began with two distinct triangles that met the SAS criterion. Now consider triangles that are not distinct and share a common vertex. The following two scenarios both show a pair of triangles that meet the SAS criterion and share a common vertex. In a proof to show that the triangles are congruent, which pair of triangles will a translation make most sense as a next step? In which pair is the next step a rotation? Justify your response.

i.



ii.



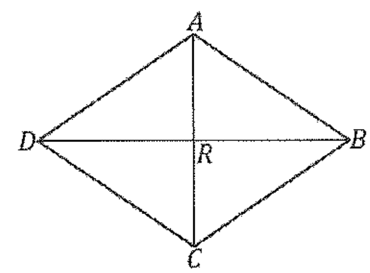
Pair (ii) will require a translation next because currently, the common vertex is between a pair of angles whose measures are unknown. The next step for pair (i) is a rotation, as the common vertex is one between angles of equal measure, by assumption, and therefore can be rotated so that a pair of sides of equal length become a shared side.

EUREKA MATH™

2. Justify whether the triangles meet the SAS congruence criteria; explicitly state which pairs of sides or angles are congruent and why. If the triangles do meet the SAS congruence criteria, describe the rigid motion(s) that would map one triangle onto the other.

   a. **Given:** Rhombus $ABCD$

   Do $\triangle ARD$ and $\triangle ARB$ meet the SAS criterion?

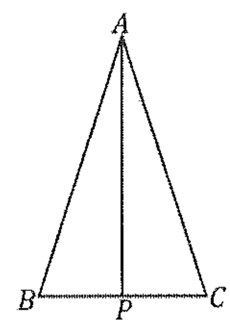


Rhombus $ABCD$	*Given*
$\overline{AR}$ and $\overline{BD}$ are perpendicular.	*Property of a rhombus*
$m\angle ARD = m\angle ARB$	*All right angles are equal in measure.*
$DR = RB$	*Diagonals of a rhombus bisect each other.*
$AR = AR$	*Reflexive property*
$\triangle ARD \cong \triangle ARB$	*SAS*

   One possible rigid motion that maps $\triangle ARD$ to $\triangle ARB$ is a reflection over the line $\overleftrightarrow{AR}$.

   b. **Given:** Isosceles triangle $\triangle$ ABC with $AB = AC$ and angle bisector $\overline{AP}$.

   Do $\triangle ABP$ and $\triangle ACP$ meet the SAS criterion?

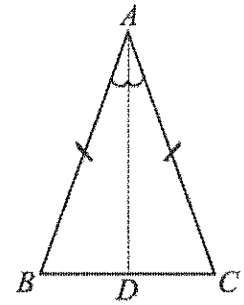


$AB = AC$	*Given*
$\overline{AP}$ is an angle bisector	*Given*
$AP = AP$	*Reflexive property*
$m\angle BAP = m\angle CAP$	*Definition of angle bisector*
$\triangle ABP \cong \triangle ACP$	*SAS*

   One possible rigid motion that maps $\triangle ABP$ to $\triangle ACP$ is a reflection over the line $\overleftrightarrow{AP}$.

Lesson 23: Base Angles of Isosceles Triangles

1. In an effort to prove that $m\angle B = m\angle C$ in isosceles triangle ABC by using rigid motions, the following argument is made to show that B maps to C:



Given: Isosceles $\triangle ABC$, with $AB = AC$

Prove: $m\angle B = m\angle C$

Construction: Draw the angle bisector $\overrightarrow{AD}$ of $\angle A$, where D is the intersection of the bisector and $\overline{BC}$. We need to show that rigid motions map point B to point C and point C to point B.

Since A is on the line of reflection, $\overleftrightarrow{AD}$, $r_{\overleftrightarrow{AD}}(A) = A$. Reflections preserve angle measures, so the measure of the reflected angle $r_{\overleftrightarrow{AD}}(\angle BAD)$ equals the measure of $\angle CAD$; therefore, $r_{\overleftrightarrow{AD}}(\overrightarrow{AB}) = \overrightarrow{AC}$. Reflections also preserve lengths of segments; therefore, the reflection of $\overline{AB}$ still has the same length as $\overline{AB}$. By hypothesis, $AB = AC$, so the length of the reflection is also equal to AC. Then $r_{\overleftrightarrow{AD}}(B) = C$.

Use similar reasoning to show that $r_{\overleftrightarrow{AD}}(C) = B$.

Again, we use a reflection in our reasoning. A is on the line of reflection, $\overleftrightarrow{AD}$, so $r_{\overleftrightarrow{AD}}(A) = A$.

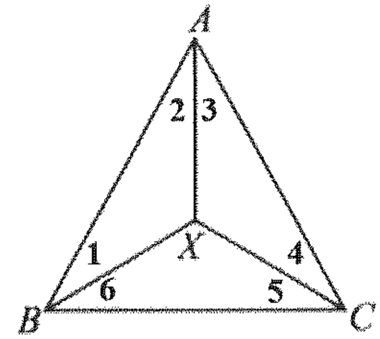
Since reflections preserve angle measures, the measure of the reflected angle $r_{\overleftrightarrow{AD}}(\angle CAD)$ equals the measure of $\angle BAD$, implying that $r_{\overleftrightarrow{AD}}(\overrightarrow{AC}) = \overrightarrow{AB}$.

> I must remember that proving this fact using rigid motions relies on the idea that rigid motions preserve lengths and angle measures. This is what ultimately allows me to map C to B.

Reflections also preserve lengths of segments. This means that the reflection of $\overline{AC}$, or the image of $\overline{AC}$ ($\overline{AB}$), has the same length as $\overline{AC}$. By hypothesis, $AB = AC$, so the length of the reflection is also equal to AB. This implies $r_{\overleftrightarrow{AD}}(C) = B$. We conclude then that $m\angle B = m\angle C$.

2. **Given:** $m\angle 1 = m\angle 2$; $m\angle 3 = m\angle 4$

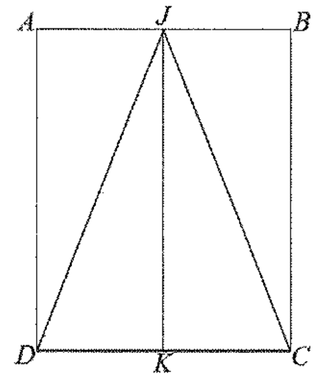
   Prove: $m\angle 5 = m\angle 6$



$m\angle 1 = m\angle 2$ $m\angle 3 = m\angle 4$	*Given*
$AX = BX$ $AX = CX$	*If two angles of a triangle are equal in measure, then the sides opposite the angles are equal in length.*
$BX = CX$	*Transitive property of equality*
$m\angle 5 = m\angle 6$	*If two sides of a triangle are equal in length, then the angles opposite them are equal in measure.*

3. **Given:** Rectangle $ABCD$; J is the midpoint of $\overline{AB}$

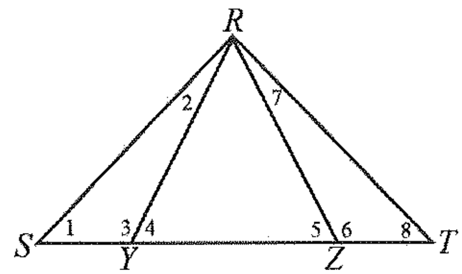
   Prove: $\triangle JCD$ is isosceles



$ABCD$ is a rectangle.	*Given*
$m\angle A = m\angle B$	*All angles of a rectangle are right angles.*
$AD = BC$	*Opposite sides of a rectangle are equal in length.*
J is the midpoint of $\overline{AB}$.	*Given*
$AJ = BJ$	*Definition of midpoint*
$\triangle AJD \cong \triangle BJC$	*SAS*
$JD = JC$	*Corresponding sides of congruent triangles are equal in length.*
$\triangle JCD$ is isosceles.	*Definition of isosceles triangle*

4. **Given:** $RS = RT; m\angle 2 = m\angle 7$

Prove: $\triangle RYZ$ is isosceles

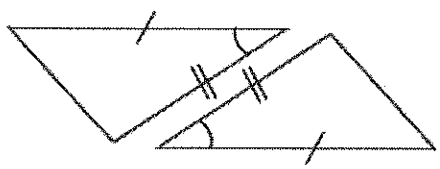


$RS = RT$	*Given*
$m\angle 1 = m\angle 8$	*Base angles of an isosceles triangle are equal in measure.*
$m\angle 2 = m\angle 7$	*Given*
$m\angle 1 + m\angle 2 + m\angle 3 = 180°$ $m\angle 6 + m\angle 7 + m\angle 8 = 180°$	*The sum of angle measures in a triangle is* $180°$.
$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 6 + m\angle 7 + m\angle 8$	*Substitution property of equality*
$m\angle 1 + m\angle 2 + m\angle 3 = m\angle 6 + m\angle 2 + m\angle 1$	*Substitution property of equality*
$m\angle 3 = m\angle 6$	*Subtraction property of equality*
$m\angle 3 + m\angle 4 = 180°$ $m\angle 5 + m\angle 6 = 180°$	*Linear pairs form supplementary angles.*
$m\angle 3 + m\angle 4 = m\angle 5 + m\angle 6$	*Substitution property of equality*
$m\angle 3 + m\angle 4 = m\angle 5 + m\angle 3$	*Substitution property of equality*
$m\angle 4 = m\angle 5$	*Subtraction property of equality*
$RY = RZ$	*If two angles of a triangle are equal in measure, then the sides opposite the angles are equal in length.*
$\triangle RYZ$ *is isosceles.*	*Definition of isosceles triangle*

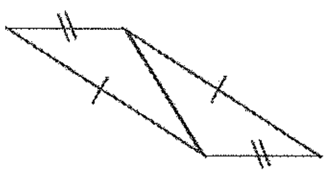
Lesson 24: Congruence Criteria for Triangles—ASA and SSS

1. For each of the following pairs of triangles, name the congruence criterion, if any, that proves the triangles are congruent. If none exists, write "none."

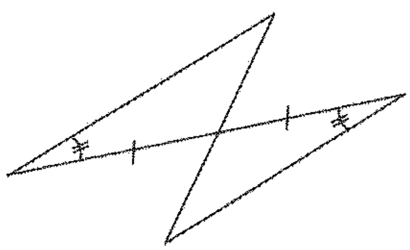
In addition to markings indicating angles of equal measure and sides of equal lengths, I must observe diagrams for common sides and angles, vertical angles, and angle pair relationships created by parallel lines cut by a transversal. I must also remember that AAA is not a congruence criterion.

a. 

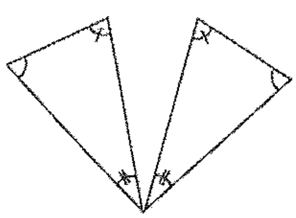
SAS

b. 

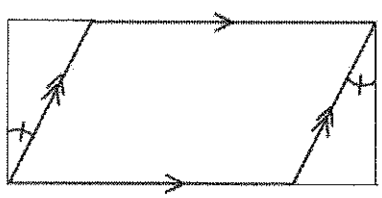
SSS

c. 

ASA

d. 

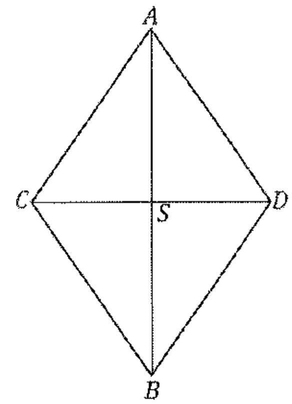
none

e. 

ASA

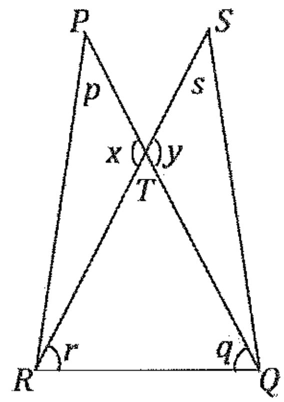
EUREKA MATH

2. $ABCD$ is a rhombus. Name three pairs of triangles that are congruent so that no more than one pair is congruent to each other and the criteria you would use to support their congruency.



Possible solution: $\triangle ACS \cong \triangle ADS$, $\triangle ACD \cong \triangle BCD$, *and* $\triangle ACB \cong \triangle ADB$. *All three pairs can be supported by SAS/SSS/ASA.*

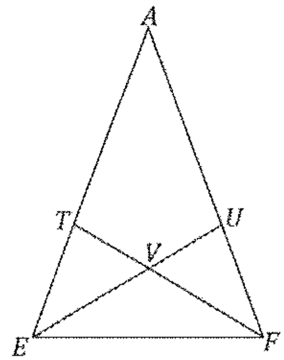
3. **Given:** $p = s$ and $PT = ST$
Prove: $r = q$



$p = s$	*Given*
$PT = ST$	*Given*
$x = y$	*Vertical angles are equal in measure.*
$\triangle PTR \cong \triangle STQ$	*ASA*
$TR = TQ$	*Corresponding sides of congruent triangles are equal in length.*
$\triangle TRQ$ *is isosceles.*	*Definition of isosceles triangle*
$r = q$	*Base angles of an isosceles triangle are equal in measure.*

4. **Given:** Isosceles $\triangle AEF$; $AT = AU$
Prove: $\angle ETF \cong \angle FUE$

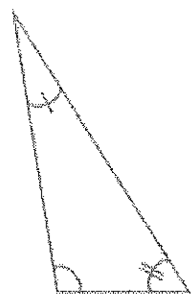
I must prove two sets of triangles are congruent in order to prove $\angle ETF \cong \angle FUE$.



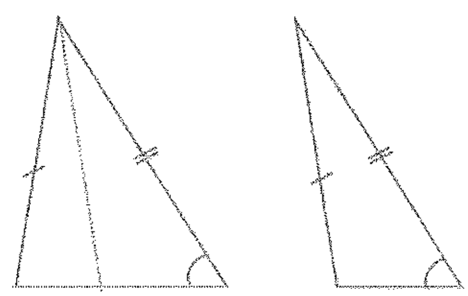
Isosceles $\triangle AEF$	*Given*
$AE = AF$	*Definition of isosceles triangle*
$AT = AU$	*Given*
$m\angle A = m\angle A$	*Reflexive property*
$\triangle AUE \cong \triangle ATF$	*SAS*
$TF = UE$	*Corresponding sides of congruent triangles are equal in length.*
$AT + TE = AE$ $AU + UF = AF$	*Partition property*
$AT + TE = AU + UF$	*Substitution property of equality*
$AT + TE = AT + UF$	*Substitution property of equality*
$TE = UF$	*Subtraction property of equality*
$EF = FE$	*Reflexive property*
$\triangle TEF \cong \triangle UFE$	*SSS*
$\angle ETF \cong \angle FUE$	*Corresponding angles of congruent triangles are congruent.*

Lesson 25: Congruence Criteria for Triangles—AAS and HL

1. Draw two triangles that meet the AAA criterion but are not congruent.



2. Draw two triangles that meet the SSA criterion but are not congruent. Label or mark the triangles with the appropriate measurements or congruency marks.



3. Describe, in terms of rigid motions, why triangles that meet the AAA and SSA criteria are not necessarily congruent.

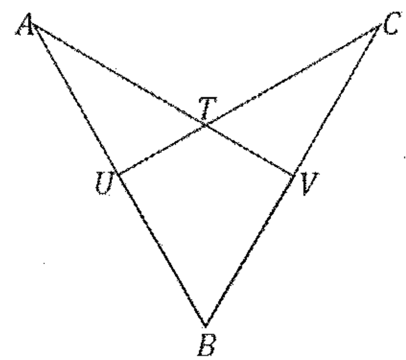
   Triangles that meet either the AAA or SSA criteria are not necessarily congruent because there may or may not be a finite composition of rigid motions that maps one triangle onto the other. For example, in the diagrams in Problem 2, there is no composition of rigid motions that will map one triangle onto the other.

4. **Given:** $\overline{AB} \cong \overline{CB}$, $\overline{CU} \perp \overline{AB}, \overline{AV} \perp \overline{CB}$,

U is the midpoint of $\overline{AB}$.

V is the midpoint of $\overline{CB}$

Prove: $\triangle ATU \cong \triangle CTV$

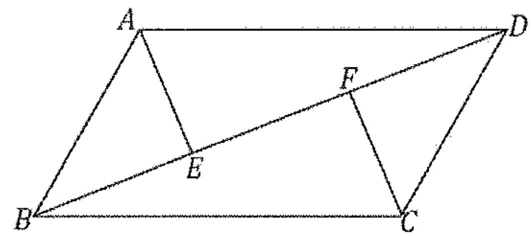


$\overline{AB} \cong \overline{CB}$	*Given*
U is the midpoint of $\overline{AB}$. *V is the midpoint of $\overline{CB}$.*	*Given*
$AB = 2AU$ $CB = 2CV$	*Definition of midpoint*
$2AU = 2CV$	*Substitution property of equality*
$AU = CV$	*Division property of equality*
$\overline{CU} \perp \overline{AB}$ $\overline{AV} \perp \overline{CB}$	*Given*
$m\angle AUT = m\angle CVT = 90°$	*Definition of perpendicular*
$m\angle ATU = m\angle CTV$	*Vertical angles are equal in measure.*
$\triangle ATU \cong \triangle CTV$	*AAS*

5. **Given:** $\overline{AE} \perp \overline{BD}$, $\overline{CF} \perp \overline{BD}$,

$AB = DC, BF = DE$

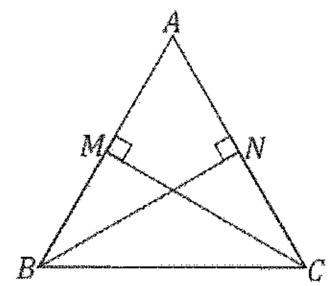
Prove: $\triangle ABE \cong \triangle CDF$



$AB = DC$	*Given*
$\overline{AE} \perp \overline{BD}$ $\overline{CF} \perp \overline{BD}$	*Given*
$m\angle AEB = m\angle CFD = 90°$	*Definition of perpendicular*
$BF = DE$	*Given*
$BF = BE + EF$ $DE = DF + FE$	*Partition property*
$BE + EF = DF + FE$	*Substitution property of equality*
$BE = DF$	*Subtraction property of equality*
$\triangle ABE \cong \triangle CDF$	*HL*

Lesson 26: Triangle Congruency Proofs

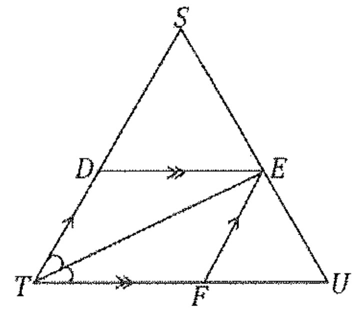
1. **Given:** $AB = AC, m\angle AMC = m\angle ANB = 90°$
   Prove: $BN = CM$



$AB = AC$	*Given*
$m\angle AMC = m\angle ANB = 90°$	*Given*
$m\angle A = m\angle A$	*Reflexive property*
$\triangle ABN \cong \triangle ACN$	*AAS*
$BN = CM$	*Corresponding sides of congruent triangles are equal in length.*

2. **Given:** $\overline{TE}$ bisects $\angle STU$. $\overline{DE} \parallel \overline{TF}$, $\overline{DT} \parallel \overline{EF}$.

Prove: $DEFT$ is a rhombus.



$\overline{TE}$ bisects $\angle STU$	*Given*
$m\angle DTE = m\angle FTE$	*Definition of bisect*
$m\angle DTE = m\angle FET$ $m\angle FTE = m\angle DET$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure.*
$TE = TE$	*Reflexive property*
$\triangle DTE \cong \triangle FET$	*ASA*
$m\angle DTE = m\angle DET$ $m\angle FTE = m\angle FET$	*Substitution property of equality*
$\triangle DTE$ is isosceles; $\triangle FET$ is isosceles	*When the base angles of a triangle are equal in measure, the triangle is isosceles*
$DT = DE$ $FT = FE$	*Definition of isosceles*
$DT = FE$ $DE = FT$	*Corresponding sides of congruent triangles are equal in length.*
$DT = FE = DE = FT$	*Transitive property*
$DEFT$ is a rhombus.	*Definition of rhombus*

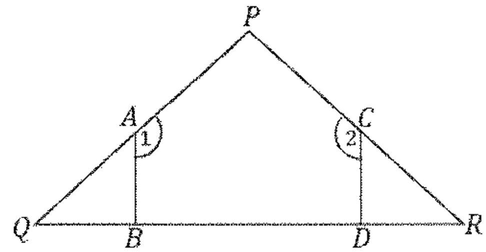
I must remember that in addition to showing that each half of $DEFT$ is an isosceles triangle, I must also show that the lengths of the sides of both isosceles triangles are equal to each other, making $DEFT$ a rhombus.

3. **Given:** $m\angle 1 = m\angle 2$

$\overline{AB} \perp \overline{QR}, \overline{CD} \perp \overline{QR}$

$QD = RB$

Prove: $\triangle PQR$ is isosceles.



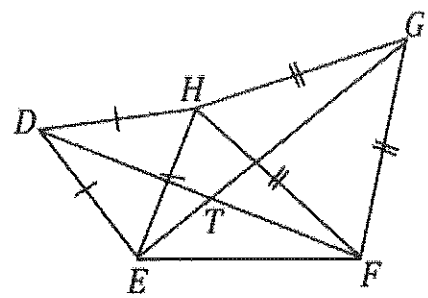
$m\angle 1 = m\angle 2$	*Given*
$m\angle QAB = m\angle RCD$	*Supplements of angles of equal measure are equal in measure.*
$\overline{AB} \perp \overline{QR}, \overline{CD} \perp \overline{QR}$	*Given*
$m\angle ABQ = m\angle CDR = 90°$	*Definition of perpendicular*
$QD = RB$	*Given*
$QD = QB + BD$ $RB = RD + DB$	*Partition property*
$QB + BD = RD + DB$	*Substitution property of equality*
$QB = RD$	*Subtraction property of equality*
$\triangle AQB \cong \triangle CRD$	*AAS*
$m\angle AQB = m\angle CRD$	*Corresponding angles of congruent triangles are equal in measure.*
$\triangle PQR$ *is isosceles.*	*When the base angles of a triangle are equal in measure, then the triangle is isosceles.*

EUREKA MATH

Lesson 27: Triangle Congruency Proofs

1. **Given:** $\triangle DHE$ and $\triangle GHF$ are equilateral triangles

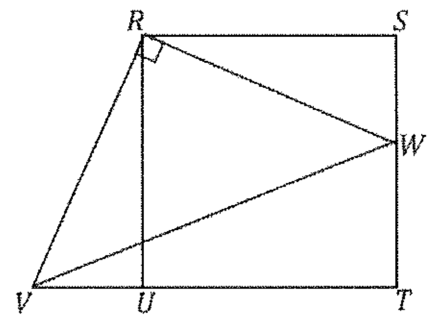
   Prove: $\triangle EGH \cong \triangle DFH$



$\triangle DHE$ and $\triangle GHF$ are equilateral triangles.	*Given*
$m\angle DHE = m\angle GHF = 60^\circ$	*All angles of an equilateral triangle are equal in measure*
$m\angle DHF = m\angle DHE + m\angle EHF$ $m\angle EHG = m\angle GHF + m\angle EHF$	*Partition property*
$m\angle DHF = m\angle GHF + m\angle EHF$	*Substitution property of equality*
$m\angle DHF = m\angle EHG$	*Substitution property of equality*
$DH = EH$ $FH = GH$	*Property of an equilateral triangle*
$\triangle EGH \cong \triangle DFH$	*SAS*

2. **Given:** $RSTU$ is a square. W is a point on $\overline{ST}$, and V is on $\overleftrightarrow{TU}$ such that $\overline{WR} \perp \overline{RV}$.

   Prove: $\triangle SRW \cong \triangle URV$

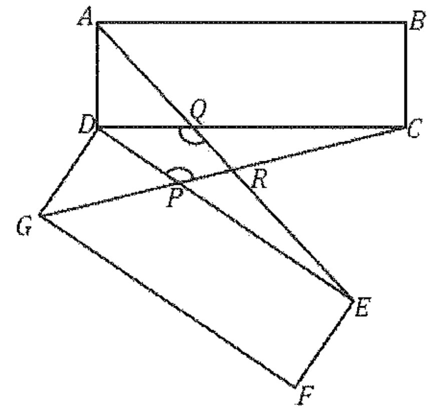


$RSTU$ is a square.	*Given*
$RS = RU$	*Property of a square*
$m\angle RUT = m\angle RSW = 90°$	*Property of a square*
$m\angle RUT + m\angle RUV = 180°$	*Angles on a line sum to 180°.*
$m\angle RUV = 90°$	*Subtraction property of equality*
$m\angle SWR = m\angle URW$	*If parallel lines are cut by a transversal, then alternate interior angles are equal in measure.*
$m\angle SRW = m\angle URV$	*Complements of angles of equal measures are equal.*
$\triangle SRW \cong \triangle URV$	*ASA*

EUREKA MATH™

3. **Given**: $ABCD$ and $DEFG$ are congruent rectangles.

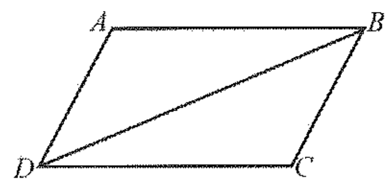
Prove: $m\angle DPR = m\angle DQR$



$ABCD$ and $DEFG$ are congruent rectangles.	*Given*
$GD = AD$ $DC = DE$	*Corresponding sides of congruent figures are equal in length.*
$m\angle GDE = m\angle ADC$	*Corresponding angles of congruent figures are equal in measure.*
$m\angle GDC = m\angle GDE + m\angle PDC$ $m\angle ADE = m\angle ADC + m\angle CDP$	*Partition property*
$m\angle GDC = m\angle ADC + m\angle PDC$	*Substitution property of equality*
$m\angle GDC = m\angle ADE$	*Substitution property of equality*
$\triangle GDC \cong \triangle ADE$	*SAS*
$m\angle GCD = m\angle AED$	*Corresponding angles of congruent triangles are equal in measure.*
$m\angle PDQ = m\angle PDQ$	*Reflexive property*
$\triangle DPC \cong \triangle DQE$	*ASA*
$m\angle DPR = m\angle DQR$	*Corresponding angles of congruent triangles are equal in measure.*

Lesson 28: Properties of Parallelograms

1. **Given**: $\triangle ABD \cong \triangle CDB$
   Prove: Quadrilateral $ABCD$ is a parallelogram

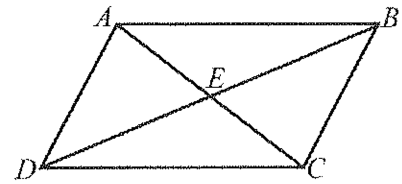


Proof:

$\triangle ABD \cong \triangle CDB$	*Given*
$m\angle ADB = m\angle CBD$; $m\angle ABD = m\angle CDB$	*Corresponding angles of congruent triangles are equal in measure.*
$\overline{AB} \parallel \overline{CD}, \overline{AD} \parallel \overline{BC}$	*If two lines are cut by a transversal such that alternate interior angles are equal in measure, then the lines are parallel.*
Quadrilateral $ABCD$ is a parallelogram	*Definition of parallelogram (A quadrilateral in which both pairs of opposite sides are parallel.)*

Since the triangles are congruent, I can use the fact that their corresponding angles are equal in measure as a property of parallelograms.

2. **Given**: $AE \cong EC; BE \cong ED$
Prove: Quadrilateral $ABCD$ is a parallelogram

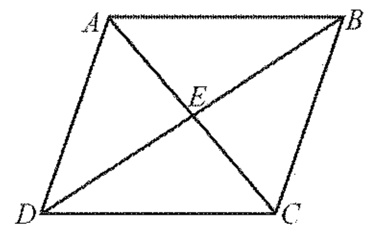


Proof:

$AE \cong CE; BE \cong ED$	*Given*
$m\angle AED = m\angle CEB$; $m\angle AEB = m\angle CED$	*Vertical angles are equal in measure.*
$\triangle AED \cong \triangle CEB$; $\triangle AEB \cong \triangle CED$	*SAS*
$m\angle ADB = m\angle CBD$; $m\angle ABD = m\angle CDB$	*Corresponding angles of congruent triangles are equal in measure*
$\overline{AB} \parallel \overline{CD}, \overline{AD} \parallel \overline{BC}$	*If two lines are cut by a transversal such that alternate interior angles are equal in measure, then the lines are parallel*
Quadrilateral ABCD is a parallelogram	*Definition of parallelogram (A quadrilateral in which both pairs of opposite sides are parallel)*

I need to use what is given to determine pairs of congruent triangles. Then, I can use the fact that their corresponding angles are equal in measure to prove that the quadrilateral is a parallelogram.

3. **Given**: Diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other; $\angle AED \cong \angle AEB$
Prove: Quadrilateral $ABCD$ is a rhombus

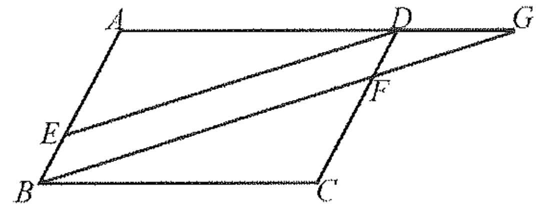


Proof:

Diagonals $\overline{AC}$ and $\overline{BD}$ bisect each other	*Given*
$AE = EC; BE = ED$	*Definition of a segment bisector*
$\angle AED \cong \angle AEB$	*Given*
$m\angle AED = m\angle AEB = 90^\circ$ $m\angle BEC = m\angle DEC = 90^\circ$	*Angles on a line sum to 180° and since both angles are congruent, each angle measures 90°*
$\triangle AED \cong \triangle AEB \cong \triangle CEB \cong \triangle CED$	*SAS*
$AB = BC = CD = AD$	*Corresponding sides of congruent triangles are equal in length*
Quadrilateral $ABCD$ is a rhombus	*Definition of rhombus (A quadrilateral with all sides of equal length)*

In order to prove that $ABCD$ is a rhombus, I need to show that it has four sides of equal length. I can do this by showing the four triangles are all congruent.

4. **Given**: Parallelogram $ABCD$, $\angle AED \cong \angle CFB$
Prove: Quadrilateral $BEDF$ is a parallelogram



Proof:

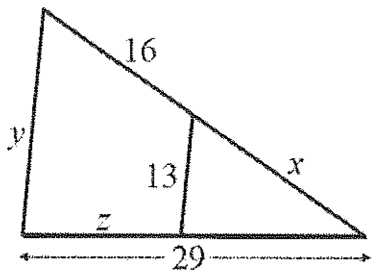
Parallelogram $ABCD$, $\angle AED \cong \angle CFB$	*Given*
$AD = BC$; $AB = CD$	*Opposite sides of parallelograms are equal in length.*
$m\angle A = m\angle C$	*Opposite angles of parallelograms are equal in measure.*
$\triangle AED \cong \triangle CFB$	*AAS*
$AE = CF$; $ED = FB$	*Corresponding sides of congruent triangles are equal in length.*
$AE + EB = AB$; $CF + FD = CD$	*Partition property*
$AE + EB = CF + FD$	*Substitution property of equality*
$AE + EB = AE + FD$	*Substitution property of equality*
$EB = FD$	*Subtraction property of equality*
Quadrilateral $BEDF$ *is a parallelogram*	*If both pairs of opposite sides of a quadrilateral are equal in length, the quadrilateral is a parallelogram*

With one pair of opposite sides proven to be equal in length, I can look for a way to show that the other pair of opposite sides is equal in length to establish that $ABCD$ is a parallelogram.

Lesson 29: Special Lines in Triangles

In Problems 1–4, all the segments within the triangles are midsegments.

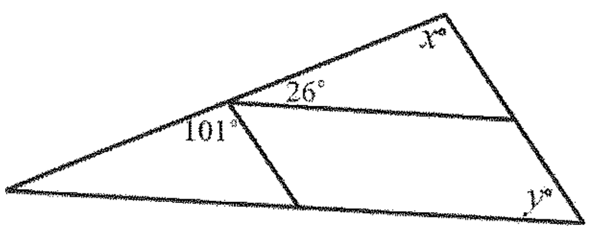
1.



$x = 16$ $y = 26$ $z = 14.5$

I need to remember that a midsegment joins midpoints of two sides of a triangle.

2.



$x = 101$ $y = 53$

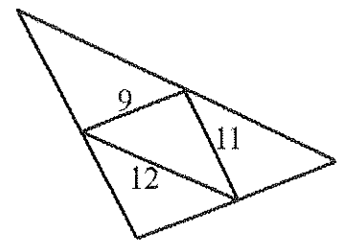
A midsegment is parallel to the third side of the triangle. I must keep an eye out for special angles formed by parallel lines cut by a transversal.

The $101°$ angle and the angle marked $x°$ are corresponding angles; $x = 101$. This means the angle measures of the large triangle are $26°$ (corresponding angles), $101°$, and $y°$; this makes $y = 53$ because the sum of the measures of angles of a triangle is $180°$.

3. Find the perimeter, P, of the triangle.

$P = 2(9) + 2(12) + 2(11) = 64$

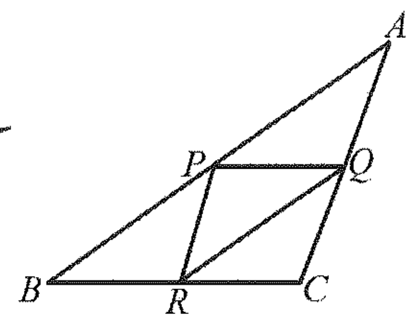
Mark the diagram using what you know about the relationship between the lengths of the midsegment and the side of the triangle opposite each midsegment.



4. State the appropriate correspondences among the four congruent triangles within $\triangle ABC$.

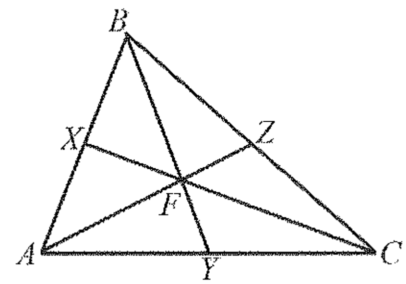
$\triangle APQ \cong \triangle PBR \cong \triangle QRC \cong \triangle RQP$

Consider marking each triangle with angle measures (i.e., $\angle 1, \angle 2, \angle 3$) to help identify the correspondences.



Lesson 30: Special Lines in Triangles

1. F is the centroid of triangle ABC. If the length of $\overline{BF}$ is 14, what is the length of median $\overline{BY}$?



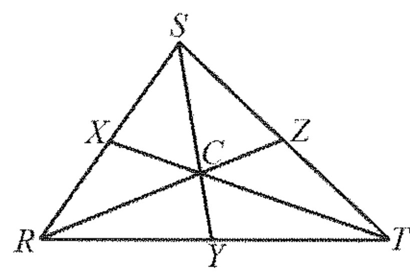
$$\frac{2}{3}(BY) = BF$$

$$\frac{2}{3}(BY) = 14$$

$$BY = 21$$

I need to remember that a centroid divides a median into two lengths; the longer segment is twice the length of the shorter.

2. C is the centroid of triangle RST. If $CZ = 9$ and $CS = 13$, what are the lengths of $\overline{RZ}$ and $\overline{SY}$?



$$\frac{1}{3}(RZ) = CZ$$

$$\frac{1}{3}(RZ) = 9$$

$$RZ = 27$$

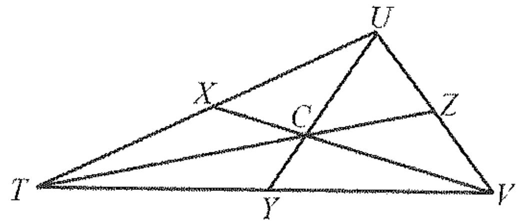
$$\frac{2}{3}(SY) = CS$$

$$\frac{2}{3}(SY) = 13$$

$$SY = \frac{39}{2} = 19.5$$

I can mark the diagram using what I know about the relationship between the lengths of the segments that make up the medians.

3. $\overline{TZ}, \overline{UY}$, and $\overline{VX}$ are medians. If $TZ = 18$, $VX = 12$ and $TU = 17$, what is the perimeter of $\triangle\ CTX$?



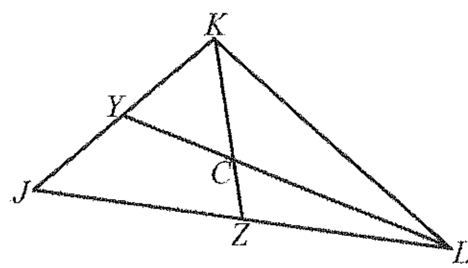
$$CT = \frac{2}{3}(TZ) = 12$$

$$CX = \frac{1}{3}(VX) = 4$$

$$TX = \frac{1}{2}(TU) = 8.5$$

Perimeter $(\triangle\ CTX) = 12 + 4 + 8.5 = 24.5$

4. In the following figure, $\triangle\ YCK$ is equilateral. If the perimeter of $\triangle\ YCK$ is 18 and Y and Z are midpoints of $\overline{JK}$ and $\overline{JL}$ respectively, what are the lengths of $\overline{YL}$ and $\overline{KZ}$?



If the perimeter of $\triangle\ YCK$ *is* 18, *then* $YC = KC = 6$.

$\overline{YC}$ and $\overline{KC}$ are the shorter and longer segments, respectively, along each of the medians they belong to.

$$\frac{1}{3}(YL) = YC$$

$$YL = 3(6)$$

$$YL = 18$$

$$\frac{2}{3}(KZ) = KC$$

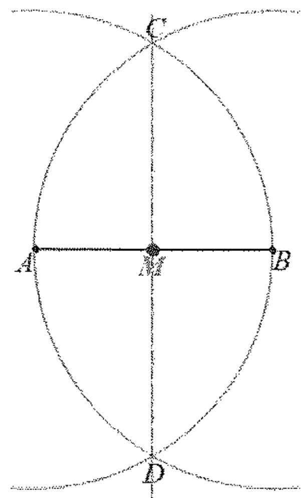
$$KZ = \frac{3}{2}(6)$$

$$KZ = 9$$

Lesson 31: Construct a Square and a Nine-Point Circle

1. Construct the midpoint of segment AB and write the steps to the construction.
   1. *Draw circle A: center A, radius AB.*
   2. *Draw circle B: center B, radius BA.*
   3. *Label the two intersections of the circles as C and D.*
   4. *Label the intersection of $\overleftrightarrow{CD}$ with $\overline{AB}$ as midpoint M.*

> The steps I use to determine the midpoint of a segment are very similar to the steps to construct a perpendicular bisector. The main difference is that I do not need to draw in $\overleftrightarrow{CD}$, I need it as a guide to find the intersection with $\overline{AB}$.



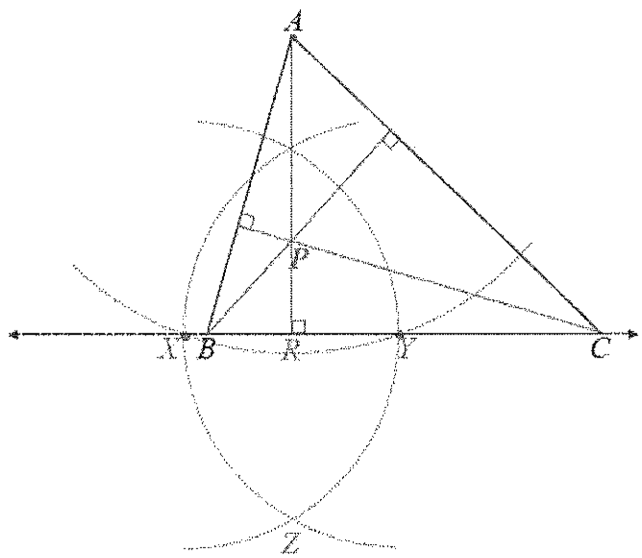
2. Create a copy of $\overline{AB}$ and label it as $\overline{CD}$ and write the steps to the construction.
   1. *Draw a segment and label one endpoint C.*
   2. *Mark off the length of $\overline{AB}$ along the drawn segment; label the marked point as D.*

EUREKA MATH

3. Construct the three altitudes of $\triangle ABC$ and write the steps to the construction. Label the orthocenter as P.

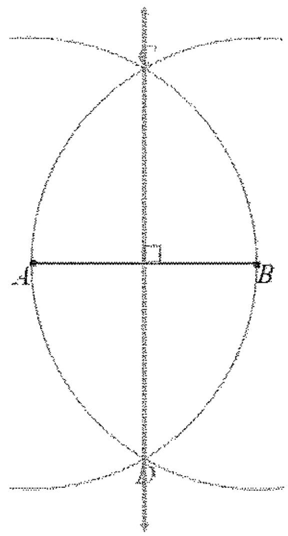
   1. *Draw circle A: center A, with radius so that circle A intersects $\overleftrightarrow{BC}$ in two points; label these points as X and Y.*
   2. *Draw circle X: center X, radius XY.*
   3. *Draw circle Y: center Y, radius YX.*
   4. *Label either intersection of circles X and Y as Z.*
   5. *Label the intersection of $\overleftrightarrow{AZ}$ with $\overleftrightarrow{BC}$ as R (this is altitude $\overline{AR}$)*
   6. *Repeat steps 1-5 from vertices B and C.*

I must remember that the intersections X or Y may lie outside the triangle, as shown in the example.



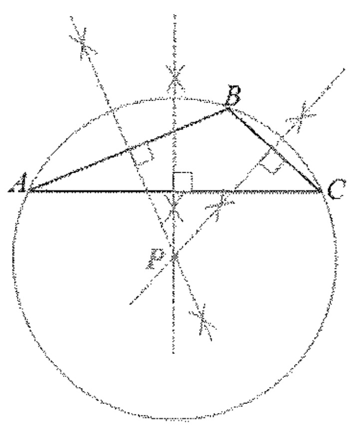
Lesson 32: Construct a Nine-Point Circle

1. Construct the perpendicular bisector of segment AB.



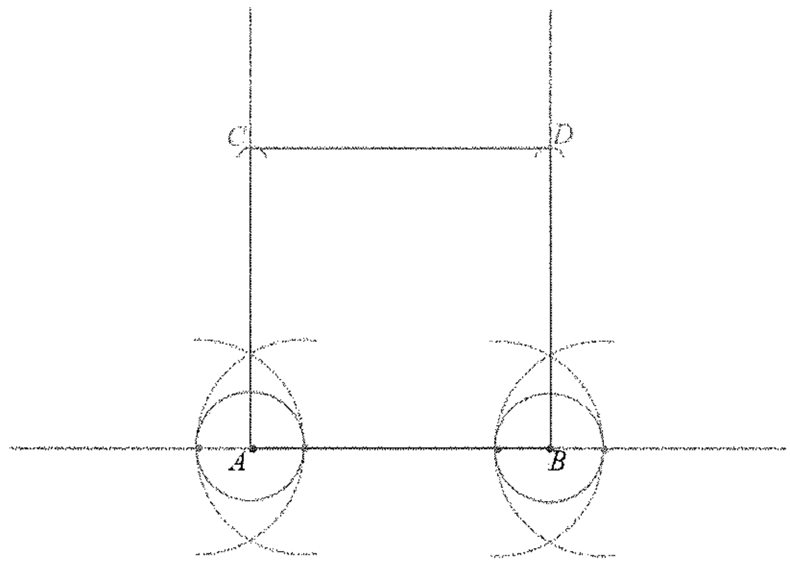
I need to know how to construct a perpendicular bisector in order to determine the circumcenter of a triangle, which is the point of concurrency of three perpendicular bisectors of a triangle.

2. Construct the circle that circumscribes $\triangle ABC$. Label the center of the circle as P.



I must remember that the center of the circle that circumscribes a triangle is the circumcenter of that triangle, which might lie outside the triangle.

3. Construct a square $ABCD$ based on the provided segment AB.



Lesson 33: Review of the Assumptions

1. Points A, B, and C are collinear. $AB = 1.5$ and $BC = 3$. What is the length of $\overline{AC}$, and what assumptions do we make in answering this question?

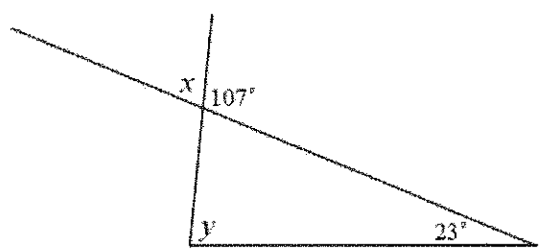
   $AC = 4.5$. *The Distance and Ruler Axioms.*

> I take axioms, or assumptions, for granted; they are the basis from which all other facts can be derived.

2. Find the angle measures marked x and y and justify the answer with the facts that support your reasoning.

   $x = 73°, y = 84°$

   The angle marked x and $107°$ are a linear pair and are supplementary. The angle vertical to x has the same measure as x, and the sum of angle measures of a triangle is $180°$.



3. What properties of basic rigid motions do we assume to be true?

   It is assumed that under any basic rigid motion of the plane, the image of a line is a line, the image of a ray is a ray, and the image of a segment is a segment. Additionally, rigid motions preserve lengths of segments and measures of angles.

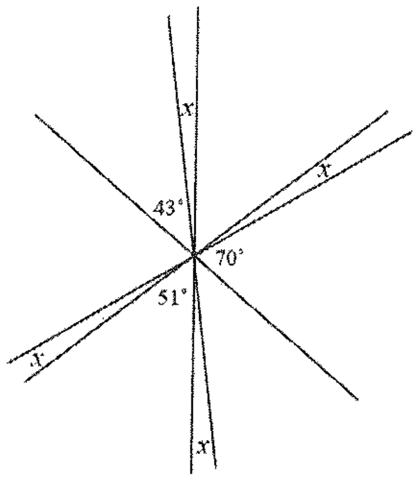
> I should remember that basic rigid motions are a subset of transformations in general.

EUREKA MATH

4. Find the measures of angle x.

The sum of the measures of all adjacent angles formed by three or more rays with the same vertex is $360°$.

$$\begin{aligned} 2(43°) + 2(51°) + 2(70°) + 4(x) &= 360° \\ 86° + 102° + 140° + 4x &= 360° \\ x &= 8° \end{aligned}$$



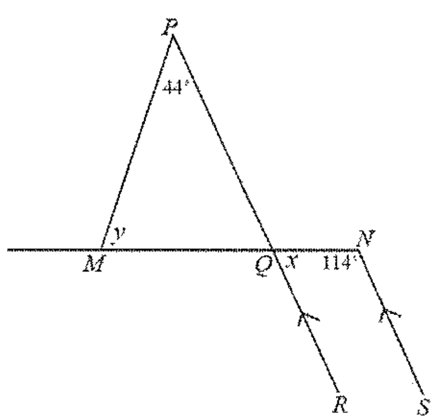
I must remember that this is referred to as "angles at a point".

5. Find the measures of angles x and y.

$x = 66°, y = 70°$

$\angle RQN$ and $\angle QNS$ are same side interior angles and are therefore supplementary. $\angle RQN$ is vertical to $\angle MQP$ and therefore the angles are equal in measure. Finally, the angle sum of a triangle is $180°$.

When parallel lines are intersected by a transversal, I must look for special angle pair relationships.



Lesson 34: Review of the Assumptions

1. Describe all the criteria that indicate whether two triangles will be congruent or not.

> I must remember that these criteria imply the existence of a rigid motion that maps one triangle to the other, which of course renders them congruent.

Given two triangles, $\triangle ABC$ and $\triangle A'B'C'$:

- *If $AB = A'B'$ (Side), $m\angle A = m\angle A'$ (Angle), $AC = A'C'$(Side), then the triangles are congruent. (SAS)*
- *If $m\angle A = m\angle A'$ (Angle), $AB = A'B'$ (Side), and $m\angle B = m\angle B'$ (Angle), then the triangles are congruent. (ASA)*
- *If $AB = A'B'$ (Side), $AC = A'C'$ (Side), and $BC = B'C'$ (Side), then the triangles are congruent. (SSS)*
- *If $AB = A'B'$ (Side), $m\angle B = m\angle B'$ (Angle), and $\angle C = \angle C'$ (Angle), then the triangles are congruent. (AAS)*
- *Given two right triangles, $\triangle ABC$ and $\triangle A'B'C'$, with right angles $\angle B$ and $\angle B'$, if $AB = A'B'$ (Leg) and $AC = A'C'$ (Hypotenuse), then the triangles are congruent. (HL)*

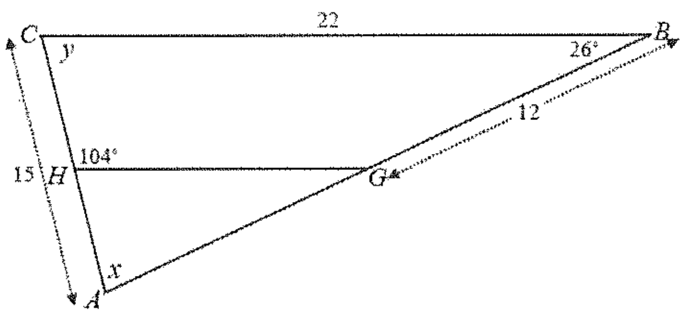
2. In the following figure, $\overline{GH}$ is a midsegment. Find x and y. Determine the perimeter of $\triangle AGH$.

I must remember that a midsegment joins midpoints of two sides of a triangle and is parallel to the third side.

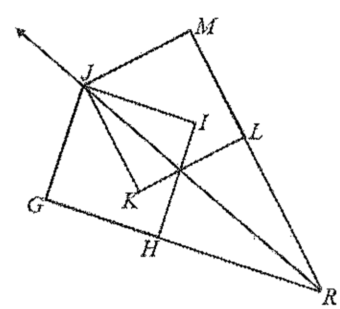
$x = 78°, y = 76°$

Perimeter of $\triangle AGH$ is:

$\frac{1}{2}(15) + \frac{1}{2}(22) + 12 = 30.5$



3. In the following figure, $GHIJ$ and $JKLM$ are squares and $GJ = JM$. Prove that $\overrightarrow{RJ}$ is an angle bisector.



Proof:

$GHIJ$ and $JKLM$ are squares and $GJ = JM$	*Given*
$\angle G$ and $\angle M$ are right angles	*All angles of a square are right angles.*
$\triangle GRJ$ and $\triangle MRJ$ are right triangles	*Definition of right triangle.*
$RJ = RJ$	*Reflexive Property*
$\triangle GRJ \cong \triangle MRJ$	*HL*
$m\angle HRJ = m\angle LRJ$	*Corresponding angles of congruent triangles are equal in measure.*
$\overrightarrow{RJ}$ is an angle bisector	*Definition of angle bisector.*

4. How does a centroid divide a median?

The centroid divides a median into two parts: from the vertex to centroid, and centroid to midpoint in a ratio of $2:1$.

The centroid of a triangle is the point of concurrency of three medians of a triangle.